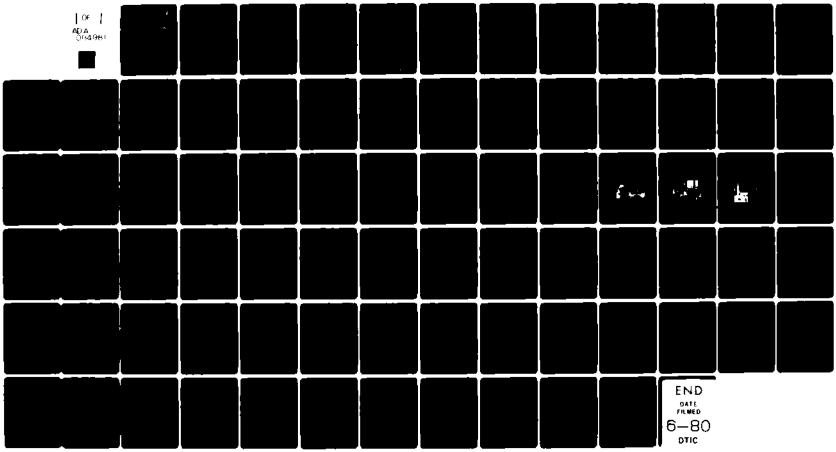
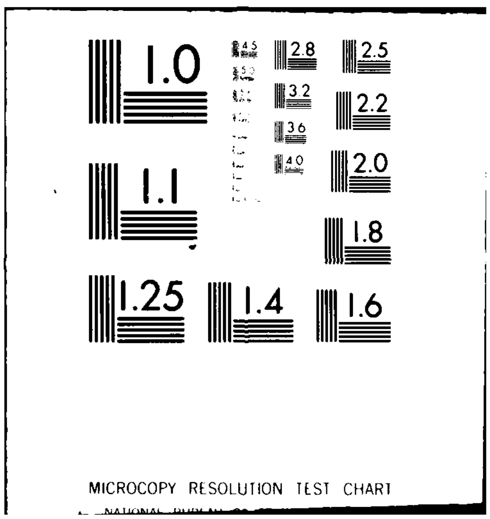
AD-A084 981 ARMY MILITARY PERSONNEL CENTER ALEXANDRIA VA F/G 13/13
CENTRIFUGAL MODELING OF AXIALLY LOADED PILES.(U)
MAY 80 J R HOUGNON

UNCLASSIFIED NL




1.0
2.8
2.5
3.2
2.2
3.6
4.0
2.0
1.1
1.8
1.25
1.4
1.6
MICROCOPY RESOLUTION TEST CHART

Centrifugal Modeling of Axially Loaded Piles

CPT James R. Hougnon
HQDA, MILPERCEN (DAPC-OPP-E)
200 Stovall Street
Alexandria, VA 22332

Final report, 8 May 1980

Approved for public release; distribution unlimited.

A thesis submitted to the University of Colorado in partial fulfillment of the requirements for the degree of Master of Science, Civil Engineering.

SECURITY CLASSIFICATION OF THIS PAGE (When Data Entered)

REPORT DOCUMENTATION PAGE

READ INSTRUCTIONS BEFORE COMPLETING FORM

1. REPORT NUMBER	2. GOVT ACCESSION NO. AD-A084981
3. RECIPIENT'S CATALOG NUMBER	
4. TITLE (and Subtitle) 6 Centrifugal Modeling of Axially Loaded Piles.	5. TYPE OF REPORT & PERIOD COVERED 9 Final Report, 8May1980
	6. PERFORMING ORG. REPORT NUMBER
7. AUTHOR(s) CPT James Robert Hougnon 10	8. CONTRACT OR GRANT NUMBER(s)
9. PERFORMING ORGANIZATION NAME AND ADDRESS Student HQDA, MILPERCEN (DAPC-OPP-E) 200 Stovall St., Alexandria, VA 22332	10. PROGRAM ELEMENT, PROJECT, TASK AREA & WORK UNIT NUMBERS
11. CONTROLLING OFFICE NAME AND ADDRESS HQDA, MILPERCEN ATTN: DAPC-OPP-E 200 Stovall St., Alexandria, VA 22332	12. REPORT DATE 11 8 May 80
	13. NUMBER OF PAGES ~~66~~ 12 79
14. MONITORING AGENCY NAME & ADDRESS(if different from Controlling Office)	15. SECURITY CLASS. (of this report) Unclassified
	15a. DECLASSIFICATION/DOWNGRADING SCHEDULE

16. DISTRIBUTION STATEMENT (of this Report)

Approved for public release; distribution unlimited.

17. DISTRIBUTION STATEMENT (of the abstract entered in Block 20, if different from Report)

18. SUPPLEMENTARY NOTES

A thesis submitted to the Faculty of the Graduate School of the University of Colorado in partial fulfillment of the requirements for the degree of Master of Science in Civil Engineering.

19. KEY WORDS (Continue on reverse side if necessary and identify by block number)

Piles, Load testing, Models, Scale models, Centrifuge, Axially loaded piles, Pile capacity, Foundations, Soil mechanics.

20. ABSTRACT (Continue on reverse side if necessary and identify by block number)

The determination of a pile's capacity involves a variety of analytical and empirical procedures, static load tests, and dynamic measurements, none of which is satisfactory in itself. Research into behavior of piles has heretofore been hampered by the excessive costs of driving and testing piles strictly for research purposes, and by the variablilty of soil conditions within a test site.

391 191

DD FORM 1 JAN 73 1473 EDITION OF 1 NOV 65 IS OBSOLETE

SECURITY CLASSIFICATION OF THIS PAGE (When Data Entered)

This thesis presents the results of research in the the behavior of axially loaded piles using centrifugal models. Scale models offer the obvious advantages of reduced cost, ease of handling, and controlled test conditions. A centrifuge is used to provide the gravity field necessary to properly represent body forces due to self-weight of the soil.

This investigation involved a series of tests on a prismatic pile and three tapered piles in a dry granular soil. The purpose of the research was to develop a system whereby piles could be driven and load tested while being rotated in a centrifuge, and, subsequently, to determine effects of taper on pile behavior.

The results of the research indicates that the system developed is capable of its intended use, but that it must be refined to obtain the results. Suggestions are made concerning modifications to improve the system.

CENTRIFUGAL MODELING OF AXIALLY LOADED PILES

by

James Robert Hougnon

B.S., United States Military Academy, 1972

A thesis submitted to the Faculty of the Graduate

School of the University of Colorado in partial

fulfillment of the requirements for the degree of

Master of Science

Department of Civil, Environmental, and

Architectural Engineering

1980

This Thesis for the Master of Science Degree by

James Robert Hougnon

has been approved for the

Department of

Civil, Environmental, and

Architectural Engineering

by

G. G. Goble

Hon-Yim Ko

Date May 8, 1980

Hougnon, James Robert (M.S., Civil Engineering)
Centrifugal Modeling of Axially Loaded Piles
Thesis directed by Professor G.G. Goble

Pile foundations have long been used to support structures in locations where the soil near the surface is inherently unsuitable for such purposes. The determination of a pile's capacity, however, involves a variety of analytical and empirical procedures, static load tests, and dynamic measurements, none of which is satisfactory in itself. Research into behavior of piles has heretofore been hampered by the excessive costs of driving and testing piles strictly for research purposes, and by the variability of soil conditions within a test site.

This thesis presents the results of research in the behavior of axially loaded piles using centrifugal models. Scale models offer the obvious advantages of reduced cost, ease of handling, and controlled test conditions. A centrifuge is used to provide the gravity field necessary to properly represent body forces due to self-weight of the soil.

This investigation involved a series of tests on a prismatic pile and three tapered piles in a dry granular soil. The purpose of the research was to develop a system whereby piles could be driven and load tested while being rotated in a centrifuge, and, subsequently, to determine effects of taper on pile behavior.

The results of the research indicates that the system developed is capable of its intended use, but that it must be refined to obtain the

best results Suggestions are made concerning modifications to improve the system.

This abstract is approved as to form and content.

Signed ______________________
Faculty member in charge of thesis

TABLE OF CONTENTS

LIST OF TABLES

LIST OF FIGURES

CHAPTER I

INTRODUCTION

The use of piles in foundations has long been one of the engineer's more practical and economical options for supporting structures in locations with poor foundation conditions. The design process for pile foundations originated as little more than a trial and error procedure. It has developed today to some rather elaborate analytical procedures, but the resulting predictions are still of sufficient uncertainty as to require full-scale load testing for practically every pile driving operation. The uncertainty stems from the tremendous variability of soils and their behavior, not only from site to site, but also within a construction site and even along the length of a pile. Adding to the unpredictability is the lack of understanding of the load carrying mechanism of piles, a fact attested to by the variety of methods used to calculate pile capacity and by the broad divergence of concepts within each of those methods.

One of the major problems in investigation of pile behavior is the tremendous cost involved. Researchers wishing to test a new theory often must find actual construction sites on which to test a limited number of piles in a limited number of conditions. Even then the costs of labor, travel, and equipment mount up rather quickly.

Testing of small scale models has been used with some degree of success in piling research, but modeling in the general sense is

limited in that the body forces of the soil mass are not properly represented. The research described by this paper involves modeling of piles using a geotechnical centrifuge. This approach is attractive since the rotation of a centrifuge creates the artificial gravity effect necessary to properly scale the body forces. The centrifuge is rapidly gaining in popularity as geotechnical engineers realize its vast potential for research of various soil mechanics phenomena. In the investigation of pile behavior, tests can be run numerous times in a variety of conditions for a fraction of the cost of a single full scale load test.

This paper will first describe the development of similitude relationships and provide a brief history of centrifugal modeling. The prediction of axial pile behavior is next dealt with in summary fashion, covering the types of piles, driving formulas, theoretical calculation of pile capacity from soil properties, and dynamic and static load measurements. Though piles can be, and quite frequently are, subjected to lateral loads, this research dealt only with axially-loaded piles. For this reason, discussion throughout this paper will be limited to axial loads.

Following this review of pile behavior, the test equipment is described. The research was conducted at the University of Colorado using a 10 g-ton capacity centrifuge. The model piles, sample containers, and measuring/recording equipment were either fabricated locally or obtained through commercial channels.

The tests were conducted on three model sizes of a prototype prismatic pile and piles of three different taper angles. It is felt that a tapered pile will provide a higher capacity than a prismatic

pile of similar dimensions. Each pile was driven and load tested in a dry granular soil at three different densities. The results of the load tests were converted to prototype scale and are presented as the load-displacement curve for each test.

In the analysis, the results are examined with respect to verification of similitude, load capacity of the piles in relation to soil density and pile taper, and comparison of test results with capacity calculated using soil properties.

CHAPTER II

HISTORY AND CONCEPT OF CENTRIFUGAL MODELS

Concept of Similitude in Modeling

The use of models is one of the engineer's most valuable tools in research and design. Modeling has long been used in numerous forms and in various engineering disciplines. Whether the model is a wind tunnel used to observe aerodynamic phenomena or a numeric model used in computer analysis of a structure, the concept is to create a system in which some property or properties behave similarly to those under investigation in the prototype. This similarity of behavior is known as similitude.

In geotechnical modeling, it is desirable to obtain similitude in stress and strain distributions in a scale model which is geometrically similar to the prototype. Under normal conditions, however, the stresses in the soil structure related to the self-weight of the soil are much smaller than in the prototype. One method of correcting this deficiency would be to use a soil skeleton material which has a density appropriately scaled up.

The centrifuge has been found to be a most useful tool in achieving the desired similitude in geotechnical modeling. In order to obtain stress similarity between model and prototype, it is necessary for the density of the soil in the model to be n times the density of the prototype soil, where the length relationship between model and

prototype is 1/n. Since density, γ, can be expressed as the product of gravity acceleration, g, and mass density, ρ, it can readily be seen that the desired density similarity can be obtained by increasing gravity n times. Using subscripts m and p to indicate model and prototype:

$$\frac{\gamma_m}{\gamma_p} = \frac{\rho_m \cdot n \cdot g}{\rho_p g} \quad . \tag{1}$$

For $\gamma_m = n\ \gamma_p$, $\rho_m = \rho_p$. Thus the prototype soil can be used in the model. The increase in gravity is achieved by rotation in the centrifuge, with the centrifugal acceleration, a, obtained by:

$$a = n\ g = r\ \omega^2 \ , \tag{2}$$

in which r = radial arm from center of rotation

ω = angular velocity of acceleration.

These relationships lead to the laws of similitude, Table I, which have been demonstrated many times. Roscoe (1968) examined the similitude laws from a theoretical basis. Some of the centrifugal testing which has demonstrated the applicability of the laws include studies of bolts in mine roofs (Panek, 1956), force relationships in penetrometer studies (Ferguson, 1980), slope stability (Kim, 1980), and time relationships in seepage analysis (Cargill, 1980).

In modeling, it is essential that the centrifugal acceleration act on the model in the same direction that gravity acts on the prototype. In a centrifuge which spins in a horizontal plane, the model normally is placed in a basket hinged so that the model ground line remains perpendicular to the resultant of the centrifuge acceleration and

TABLE I
Similitude laws

QUANTITY	PROTOTYPE	MODEL
Length	n	1
Area	n^2	1
Volume	n^3	1
Velocity	1	1
Acceleration	1	n
Mass	n^3	1
Force	n^2	1
Energy	n^3	1
Stress	1	1
Strain	1	1
Mass Density	1	1
Time (dynamic problems)	n	1
Frequency	1	n

earth's gravity. It should be noted, also, that earth's gravity exerts an influence on the model. This influence need be considered, however, only for models spun at very slow speeds. At a centrifugal acceleration of 5 g, the resultant acceleration is less than 2% greater than the horizontal component.

History of the Centrifuge in Geotechnical Research

The centrifuge was apparently first used for modeling soil structures in the early 1930's. Bucky (1931, 1935, 1938) investigated the behavior of models in a centrifuge by photo-elastic methods. He then applied his results to research in mining structures. At approximately the same time, a group of engineers in the Soviet Union began investigating foundation deformations by centrifugal modeling (Pokrovsky and Fedorov, 1936). In the late 1960's the University of Cambridge built its first centrifuge and began extensive research of soil-mechanics problems (Roscoe, 1968), as did Okasa City University in Japan (Makasa and Takada, 1973). In the last fifteen years, the centrifuge has enjoyed a surge in use. There are currently at least four centrifuges in the United States and several in Europe used for geotechnical research in addition to those already mentioned. As the methods are improved for building and instrumenting the models, the engineering community will find the centrifuge used increasingly for investigation of geotechnical phenomena as well as in the analysis and design for particular structures.

CHAPTER III

DETERMINATION OF PILE CAPACITY

The use of piles to provide a suitable foundation for structures in locations where the soil at or near the surface is unsuitable has been common since at least 59 A.D. (Peck, Hanson and Thornburn, 1953). Much progress has been made in the development of piles from the early wooden piles to today's deep piles used in off-shore drilling. Nevertheless, the prediction of pile capacity by analytical means is an inexact science, at best.

Generally speaking, a pile is a structural member placed in the ground to transfer load through a weak soil stratum to underlying competent strata, or to make better use of the strength of a soil layer. Piles can be classified by material, by shape, and by placement method. In the first portion of this chapter, the various piles types will be identified, and the remainder of the chapter will be devoted to a brief discussion of the various methods used to determine the load carrying capacity of a pile.

Types of Piles

Materials

The first material used for piles, and until this century the most common one, was wood. The primary advantage of timber piles is their

availability. Wood is particularly stable when entirely submerged, but susceptible to rot above the water unless protected by some form of chemical treatment. Late in the 19th century, concrete came into use as a pile material. Today both conventionally reinforced and prestressed concrete piles are frequently used. The need for deeper foundations has led to the use of steel in piles. Composite piles are those using two or more of these materials, for example a concrete-filled steel pipe.

Cross section

Piles are used in a variety of cross-sectional shapes, of which some of the more common are shown in Figure 1. Wood, steel pipe, and many concrete piles have round cross-sections. Rectangular concrete piles can also be frequently found. H-shapes are very common among steel piles. These are normally the sections designated HP by the American Institute for Steel Construction (AISC, 1970), although wide-flange (W) and standard (S) shapes may be used. Monotube is a trade name for a type of section which was originally fabricated as a lamp post. This particular shape has been found to have many favorable characteristics for use as piles.

Longitudinal shape

Piles often are prismatic (have a constant cross section throughout the length), but tapered piles have been found to provide greater bearing capacity for a given length. Tapers from 0.03 in/ft to 0.40 in/ft have been used (Peck, Hanson, and Thornburn, 1953). A variation is the step-tapered pile, made of sections without taper, but each section with a diameter smaller than the one above it. Point bearing

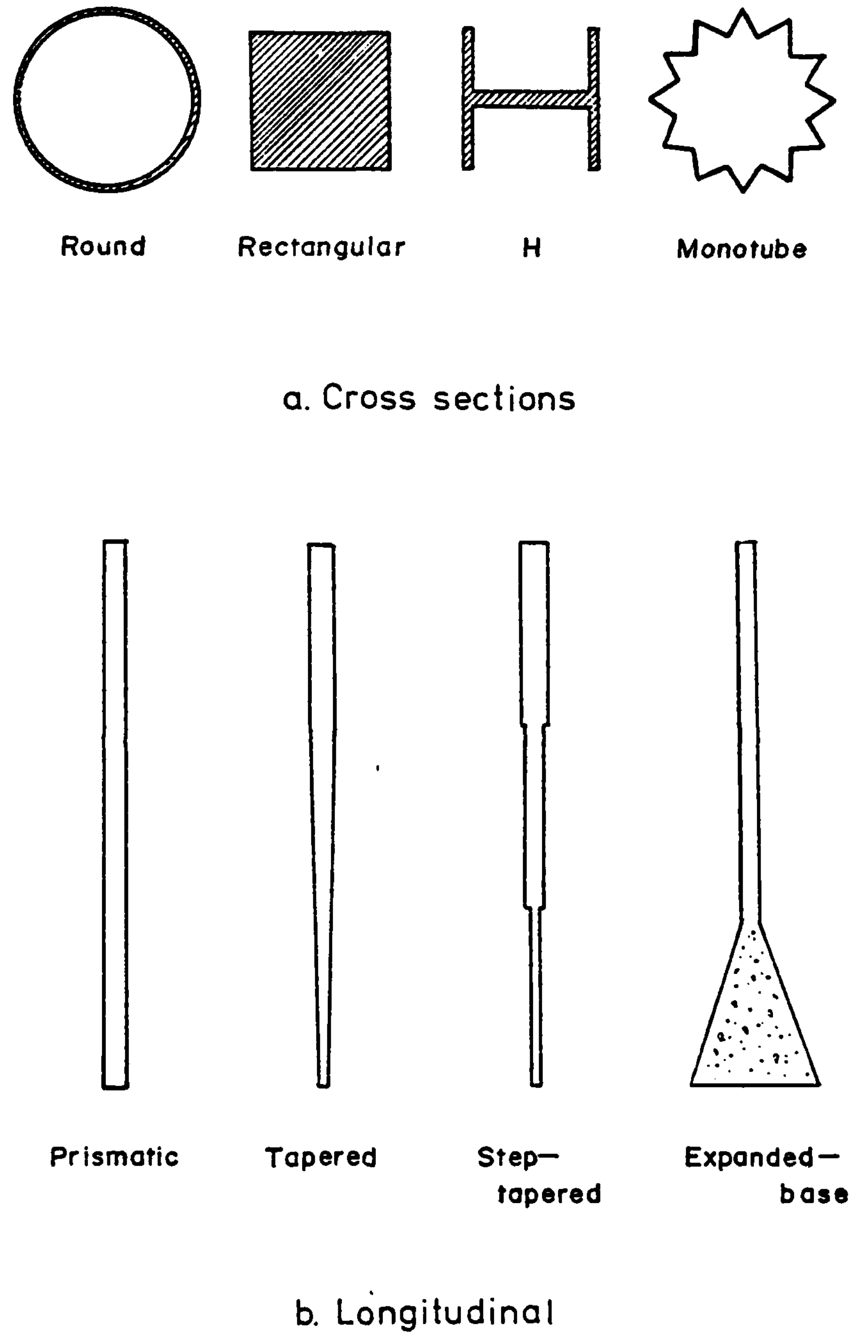


Fig. 1. Common pile shapes

piles, particularly where the bearing stratum is thin enough to be susceptible to punch-through, may be built with an expanded base. In these cases, a volume of soil is either bored, jetted, or simply pushed out of the way to allow for an extra bulb or bell-shaped mass of concrete at the base of the pile.

Placement method

Piles are installed in the ground by pushing, driving, or boring. Pushing is often the most desirable method because it involves the least disturbance of the surrounding soil. It is, however, generally the least practical because of the high static force necessary to place the pile.

The most common method of placement is driving, the pounding of the pile into place with a hammer. In medieval times, the hammers were swung by men. Modern equipment is operated by steam, compressed air, or diesel. Single-acting hammers lift the ram to a specified height and let it fall. Double-acting hammers lift the ram and force it back down. The energy of the hammer may be delivered either to the top of the pile or to a mandrel inside the pile. A third method of placement is boring or augering. A hole is bored in the ground and usually filled with concrete. A casing may be used if the soil conditions require it. Bored piles are commonly referred to as caissons.

Driving formulas

In design of pile foundations, the engineer is, of course, interested in the capacity of the pile. Early methods of predicting capacity were based on the hammer energy used to drive the pile. Many such

formulas exist and are known as dynamic pile formulas or driving formulas. Many engineers put little faith in the driving formulas because they have some serious shortcomings in their development. For example, the energy terms do not fully account for the hammer velocity or for the influence of the cushion between hammer and pile. In some cases there is no alternative but to use a formula in spite of its unreliability.

Two commonly-used driving formulas

The earliest and probably widest-known driving formula is the Engineering News formula:

$$Q_a = \frac{2E}{s+0.1}, \tag{3}$$

where Q_a = allowable pile load in pounds

E = energy per blow in foot-pounds

s = average penetration in inches per blow for the final 6 inches.

A formula adopted by the Boston Building Code in 1964 is a more refined version:

$$Q_a = \frac{1.7E}{s+0.1\ w_p/w_r} \tag{4}$$

where w_p = weight of the pile and other driven parts

w_r = weight of the striking part of the hammer.

Other dynamic formulas can be found in Whitaker (1976) and Chellis (1944 and 1951).

Calculation of Pile Capacity from Soil Properties

Load transfer

The load carried by a pile is transferred to the soil by either end-bearing, skin friction, or a combination of the two. The relationship between the two types of transfer for a given pile is known as the load transfer function. The load transfer function is highly dependent on the characteristics of the soil strata, the type of pile, and even the amount of load. In computing the ultimate load, each of the two components is calculated separately, and the two are then added:

$$Q_a = Q_p + Q_f \qquad (5)$$

where Q_p = end-bearing or point resistance

Q_f = skin friction or mantle resistance.

Formula for end bearing

The point resistance is the product of the area of the pile tip and the bearing capacity of the soil. The bearing capacity is calculated in essentially the same manner as that of a shallow foundation:

$$Q_p = A_p \left(\bar{c}N_c + \frac{\gamma B N_\gamma}{2} + \gamma d N_q \right) \qquad (6)$$

in which $\bar{c}$ = effective cohesion

γ = unit weight of soil

B = footing width

d = depth of footing below ground level

N_c, N_γ, N_q = bearing capacity factors based on the friction angle

For piles, $\gamma B/2$ is small compared to the other terms, so it is neglected.

For a pile in sand, $\bar{c} = 0$ and $\phi = \bar{\phi}$. The point resistance is thus:

$$Q_p = A_p \gamma d N_q \qquad (7)$$

with A_p = area of pile tip

d = depth of penetration.

The bearing capacity factor, N_q, is reasonably well-defined for shallow foundations. In the case of piles, however, most investigators see N_q as being higher than for the shallow foundation case. Figure 2 shows various interpretations of N_q as a function of friction angle for deep circular foundations (Lambe and Whitman, 1969).

For non-free draining soils (clays), $\phi = 0$ so that $N_q = 1$. For this case, c is the average undrained shear strength s_u, and the point resistance becomes:

$$Q_p = A_p\ (s_u N_c + \gamma d). \qquad (8)$$

For values of N_c, see Lambe and Whitman, 1969.

<u>Formula for mantle resistance</u>

Ideally, the mantle resistance of a pile is found by the integral:

$$Q_s = \int_L (a_s s_s)(d\ell), \qquad (9)$$

where a_s = surface area of the pile as a function of length

s_s = unit shaft resistance as a function of length.

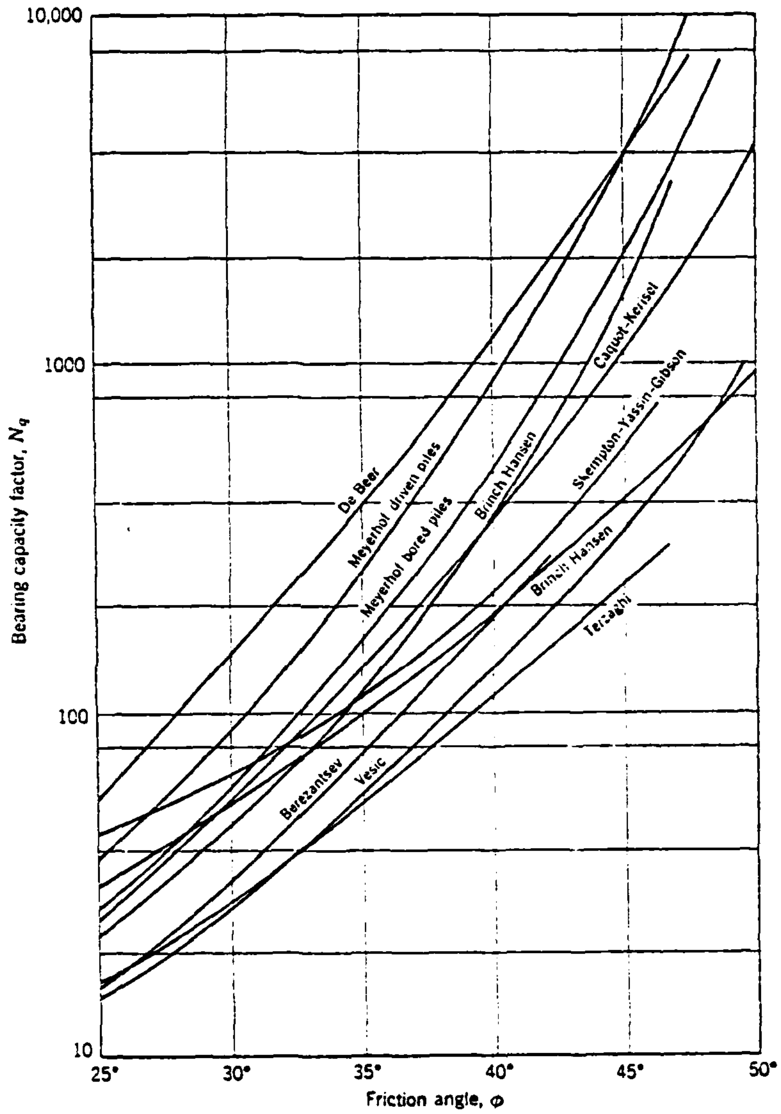


Fig. 2. Bearing capacity as a function of friction angle (Lambe and Whitman, 1969)

In practice, the shaft resistance is usually found to be a linear function, with a_s constant for a prismatic pile. Thus, the friction portion of the load capacity is:

$$Q_s = A_s L s_s, \tag{10}$$

in which s_s is taken for mid-depth of the pile.

The value used for the unit skin friction has been the subject of much research. Lambe and Whitman (1969) suggest that for free draining soils,

$$s_s = K\gamma d \tan \bar{\phi}, \tag{11}$$

where $K = .5$ to 3.

For non-free draining soils:

$s_s = s_u$, the average undrained shear strength.

Total pile capacity

The load a pile in sand is capable of carrying is:

$$Q = A_p \gamma d N_q + A_s K \gamma \frac{d}{2} \tan \bar{\phi} \ . \tag{12}$$

For a pile in clay, the capacity is found by:

$$Q = A_p \ (s_u N_c + \gamma_d) + A_s s_u. \tag{13}$$

Calculation of capacity for the prototype pile

To show the range of results which can be obtained from equations 12 and 13, the capacity of the prototype pile will be calculated twice. In one case minimum values will be used for K, $\bar{\phi}$, and N_q, and in the

other case maximum values will be used for K and $\bar{\phi}$ and an intermediate value will be selected for N_q.

The pile is a 16" diameter solid steel pile, 45' long. It is driven into dry sand with γ_d = 109.5 pcf and $\phi = 43.5^\circ$.

$$A_p = 1.40 \text{ ft}^2$$

$$A_s = 188.5 \text{ ft}^2$$

$$\gamma_d = 4928 \text{ pcf}$$

$$s_s = K\gamma\frac{d}{2} \tan \bar{\phi} \text{ (at mid-depth)}$$

For the minimum case, K is taken as .5, and ϕ as 30°, which is an average friction between steel and sand. For N_q, a solution based on Rankine wedges will be used (Lambe and Whitman, 1969):

$$N_q = \left(\frac{1+\sin\phi}{1-\sin\phi}\right)^2 .$$

While this value is not widely accepted, it is useful for comparison.

For this case:

$$Q = (1.40)(4928)(29.3) + (188.5)(.5)(4928)(\tfrac{1}{2})(\tan 30)$$

$$= 202{,}147 + 134{,}079$$

$$= 336{,}226 \text{ lb}$$

$$= 168 \text{ tons.}$$

For the second situation, K is taken as 3, $\bar{\phi}=\phi=43.5^\circ$, and a value of N_q = 410 is selected from Figure 2. For these values:

$$Q = 2{,}828{,}672 + 1{,}322{,}280$$

$$= 4{,}150{,}952$$

$$= 2{,}075 \text{ tons}$$

Load Testing

As can be seen from the preceding example, the prediction of capacity based on soil properties can lead to a wide variety of solutions. The variety of thinking on the various parameters and the variability of soil do not create the type of reliability that a designer would like to have in his foundation, nor do the pile driving formulas discussed earlier produce such confidence. In order to verify the design it is necessary to perform load tests on at least a portion of the piles in a foundation.

Dynamic measurements

Several advances have been made in recent years in the measurement of pile capacity by dynamic response. This method is based on analysis of the shock wave propagated through the pile and soil when the pile is struck by the hammer. The results are used to predict driving stresses in the pile as well as to determine an ultimate load. Several systems have been developed to analyze the wave motion, for example, Wave Equation Analysis of Pile Driving-WEAP (Goble and Rausche, 1976) and the Texas Transportation Institute program - TTI (Sampson, Hirsch, and Lowery, 1963).

Static load testing

A static load test is conducted by placing incremental loads on a driven pile and measuring the displacement of the top of the pile.

The load for this type of test can be applied by placing dead weight on a platform on top of the pile or by loading the pile with a jack. Figure 3 shows one method of loading by a hydraulic jack.

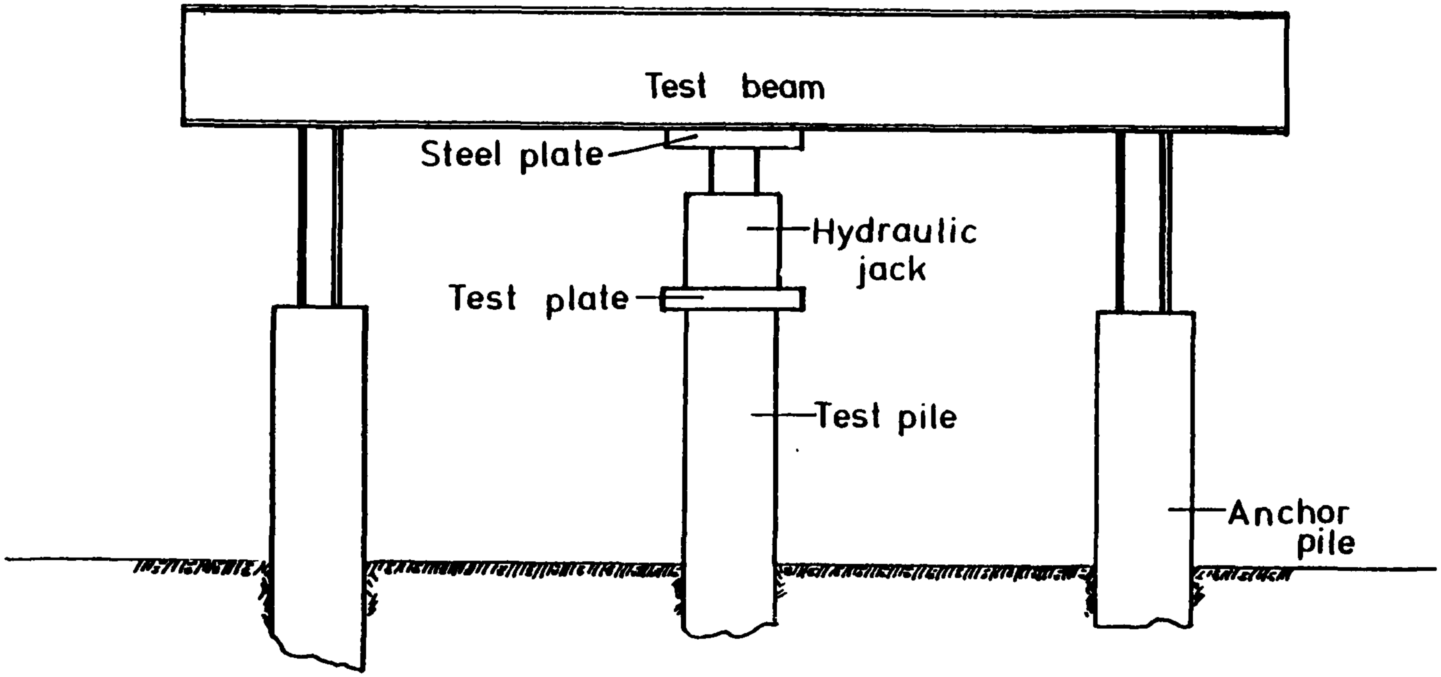


Fig. 3. Static load test setup

The two common methods of static load testing are the maintained load (ML) and the constant rate of penetration (CRP) tests. For the ML test, a load increment is applied and maintained until the displacement has substantially ceased. Then the load and displacement are measured and plotted. In the CRP test, the applied load is controlled so that the pile penetration speed remains constant. Figure 4 shows typical results of both tests on the same pile.

Determination of design load from load test results

Once a load-displacement curve has been obtained from a load test, some criterion must be applied to choose the ultimate load. Since there typically is not a sharp breaking point in the load test curve, one cannot simply pick out the design load by visual inspection. The strength of the soil is mobilized by displacement of the pile. The displacement necessary to mobilize the skin friction is principally dependent on soil properties, while the displacement required to obtain full bearing capacity is subject to the pile tip dimensions. Numerous criteria have been developed based on the load test curve properties (Vesic, 1975). Several of the criteria establish the ultimate load as that which corresponds to a specified displacement. Such criteria have not been widely applied. One criterion widely accepted by the industry was proposed by Davisson (1972), and is based on the pile stiffness and the tip diameter. According to his proposal, a line with a slope, K, equal to the pile stiffness is drawn on the load displacement curve, with an offset of δ_A.

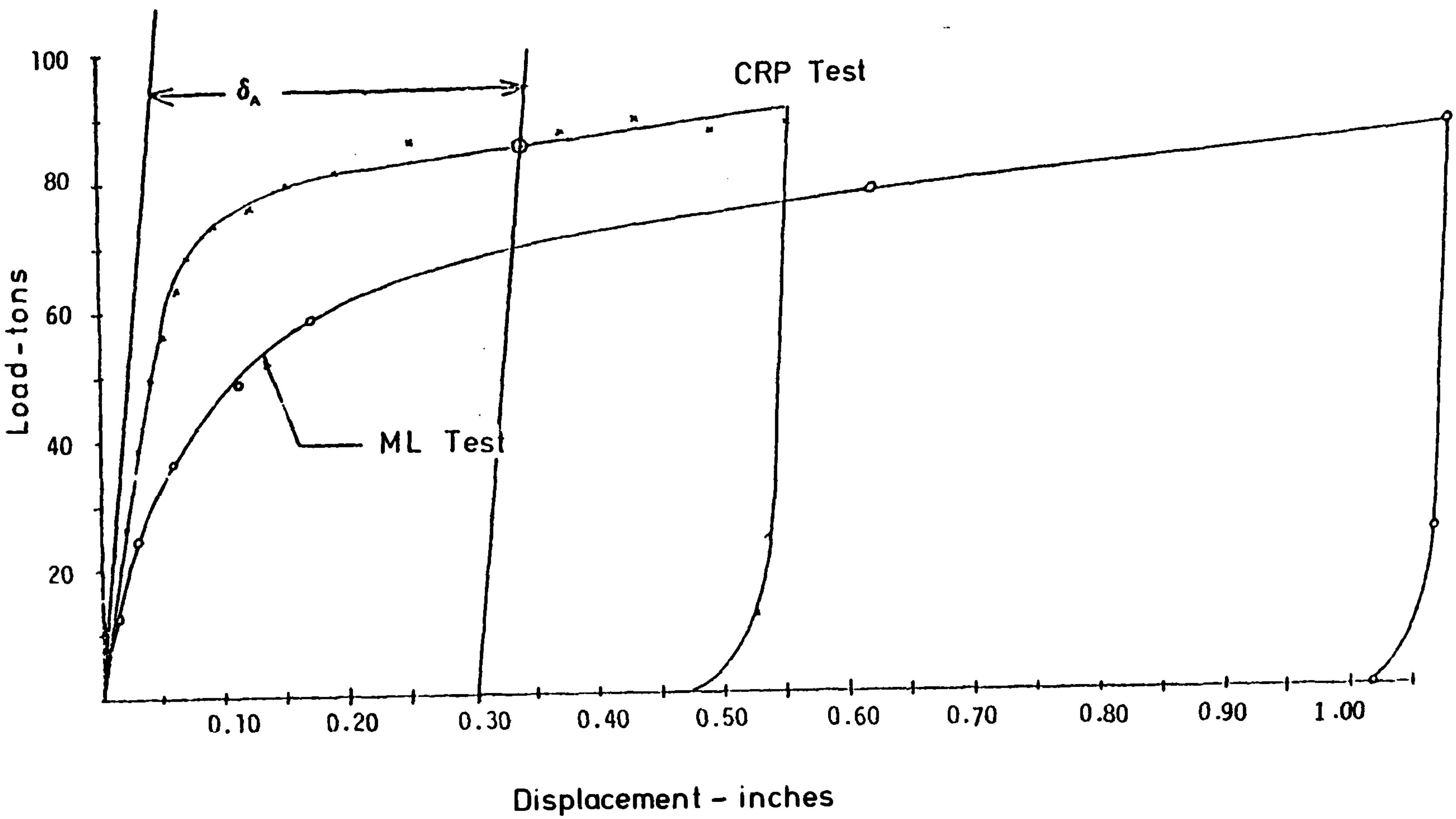


Fig. 4. Typical load test results (Goble and Likins, 1973)

$$K = \frac{AE}{L} \tag{14a}$$

and

$$\delta_A = 0.15 + 0.1\,\frac{D}{12}\,, \tag{14b}$$

where A = cross-sectional area of the pile

E = modulus of elasticity for the pile

L = overall pile length

δ_A = offset in inches

C = pile tip diameter in inches.

This line has been shown on Figure 4.

CHAPTER IV

TEST EQUIPMENT

Centrifuge

General description

The centrifuge (Figure 5) used for this investigation was acquired by the University of Colorado in 1978. It is a Genesco G-Accelerator Model 1230-5, capable of spinning at 470 rpm. The payload capacity for this machine is 10 g-ton, meaning it can accelerate a package of 200 pounds up to a force level of 100 g. The centrifuge is housed in a 3-foot high, 9-foot diameter steel cylinder, with the controls and the drive pump mounted separately.

Drive and control systems

The centrifuge is driven by a rotary oil-gear system. The oil for this system is pressurized by a 2500 psi capacity centrifugal pump, which is powered by a 25 hp electric motor (Figure 6). A solenoid valve allows oil to flow into the drive system and provides a means for quick shut-down in case of emergency. A second valve can be adjusted to regulate oil flow and centrifuge speed.

The operation of the centrifuge is controlled from a box located adjacent to the pump (Figure 7). This box contains a toggle switch and the start-stop buttons for the electric motor, a toggle switch for the solenoid valve, a crank wheel to operate the speed control valve on

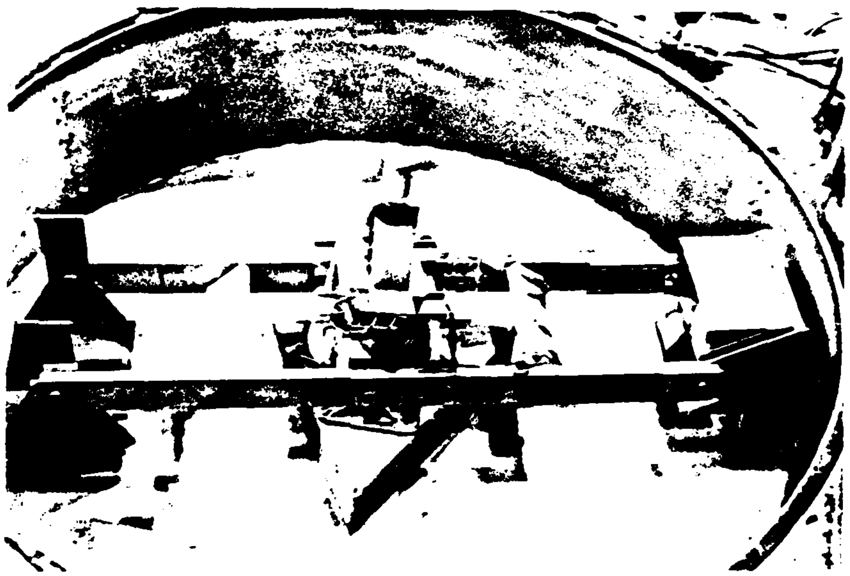


Fig. 5. University of Colorado geotechnical centrifuge

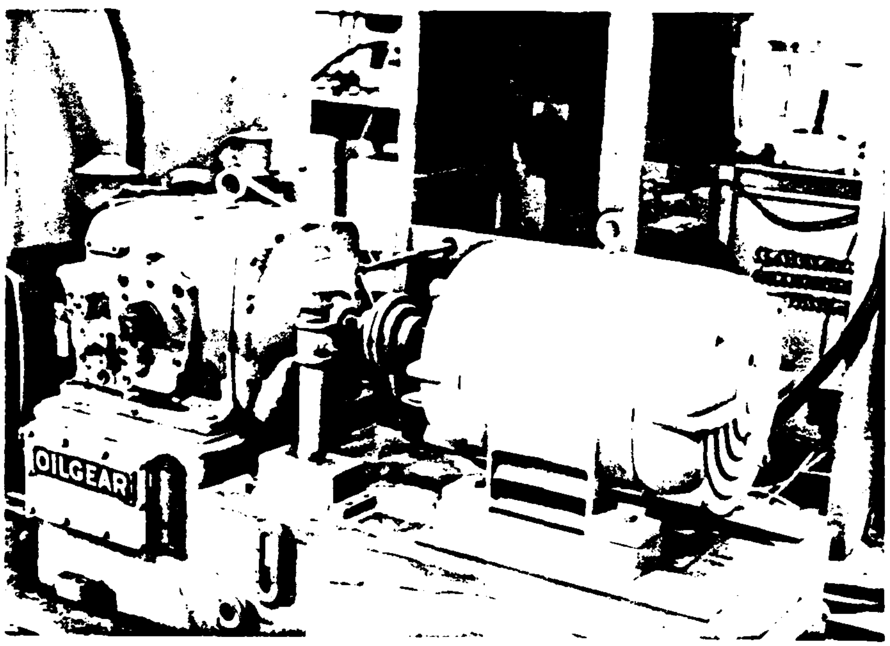


Fig. 6. Centrifuge drive pump and motor

Fig. 7. Centrifuge controls

the pump, and a tochometer. Also mounted on the control box are terminals for electronics equipment, several 110 volt AC electric outlets, and an electric circuit breaker for the entire assembly.

Arms

A pair of structural aluminum arms are mounted opposite each other in the centrifuge. The arms rotate about a fixed axis within the containment cylinder. At the end of each pair of arms is a swinging basket. The sample under study is normally mounted on the floor of the basket, which is approximately 18" square. The basket sides are 23.5" high and are attached to pivot points centered 11.5" above the floor of the baskets. The bottoms of the baskets are weighted so that under a centrifugal force the sample will swing out and up. The effect is to keep the resultant centrifugal force perpendicular to the model ground line at all times. Threaded holes are provided on the upper portion of the basket sides for mounting test equipment. The two baskets are identical to allow for simultaneous duplication of tests. Normally, however, as with the present investigation, a test package is placed in one basket, and the other end is counter-weighted to balance the arms.

Slip rings

Fifty six electrical slip rings are mounted on a rotating shaft extending from the main axis of the centrifuge. The slip rings are used to power electric equipment and to monitor electronic measuring devices on the test package from outside the machine. A 5-amp capacity 110 v. AC electric circuit is carried into the machine through three of the slip rings.

At the top of the shaft is a pair of fluid slip rings rated at 3000 psi. For this investigation, these rings were used to pass hydraulic fluid to activate the pile driving and testing mechanism.

Recent research on this machine

In the past year, the University of Colorado's centrifuge has been used extensively for basic as well as applied research. Research done by graduate students includes investigations of cone penetrometers (Ferguson, 1980), slope stability (Kim, 1980), and water seepage through earth embankments (Cargill, 1980). The Martin Marietta Corporation also has used this machine to study the breakout capability of the MX missile.

Limitations

A significant shortcoming with this particular centrifuge is its small size. The radial distance from the axis to the pivots of the basket is 42". Most test packages thus have a radial arm of 46-50 inches. Since centrifugal force varies directly with radius, large differences in the force level can occur over a small sample depth. For instance, consider a model pile 10" long with its center of gravity at 48", spun to 50 g. The centrifugal force at the top of the pile (radius of 43") is only 45 g, while the tip (53" radius) feels 55 g. If the same model were spun to 50 g in a centrifuge with the center of the pile at 96", the resulting top and tip force levels would be 47 g and 53 g respectively.

Another limitation imposed by the size of the machine is the package size available. The specimen is limited to approximately

18" x 18" x 12", which is rather small for most large scale modeling. However, this is not a problem for pile models.

Piles

Size of model piles

The sizes of the piles were dictated by the choice of prototype pile and scaling factors. The prototype was chosen as a 16" diameter solid steel pile, 45' in length. Scale factors used were 50, 70, and 100, resulting in the model sizes shown in Table II.

TABLE II
Model pile sizes

SCALE FACTOR	LENGTH (FEET)	DIAMETER (INCHES)
1 (prototype)	45	16
50	0.90	0.32
70	0.64	0.23
100	0.45	0.16

Construction

The piles were made of standard sizes of drill rods. Due to the great variety of drill rod sizes, it was possible to select diameters which were the desired sizes.

A 3/8" steel ball was silver soldered to the top of each pile to allow for a pin-type connection with the driving and loading mechanisms.

Tapers

Tests were conducted on four pile types (prismatic and three taper angles) at the three scale sizes, for a total of twelve piles. Each of the tapered piles was machined from full diameter down to half diameter. The angles used were 0.09, 0.14, and 0.40 inches/foot. The resulting taper lengths are shown in Table III.

TABLE III

Tapered pile dimensions

	LENGTH OF TAPER (feet)		
TAPER ANGLE (in/ft)	0.09	0.14	0.40
Scale factor			
1 (prototype)	45	28.6	10.0
50	.90	.57	.20
70	.64	.41	.14
100	.45	.29	.10

Driving and Load Test Mechanism

Container

The soil container (Figure 4) was a 7.2" inside diameter cylindrical aluminum mold, one foot in depth. The mold was mounted on a 17" x 18" x 1/2" aluminum plate bolted to the floor of the centrifuge basket. Three such containers were made.

Driving head

The piles were driven hydraulically using a 4-ton capacity double-acting cylinder and a rack and gear system (Figure 8). The cylinder

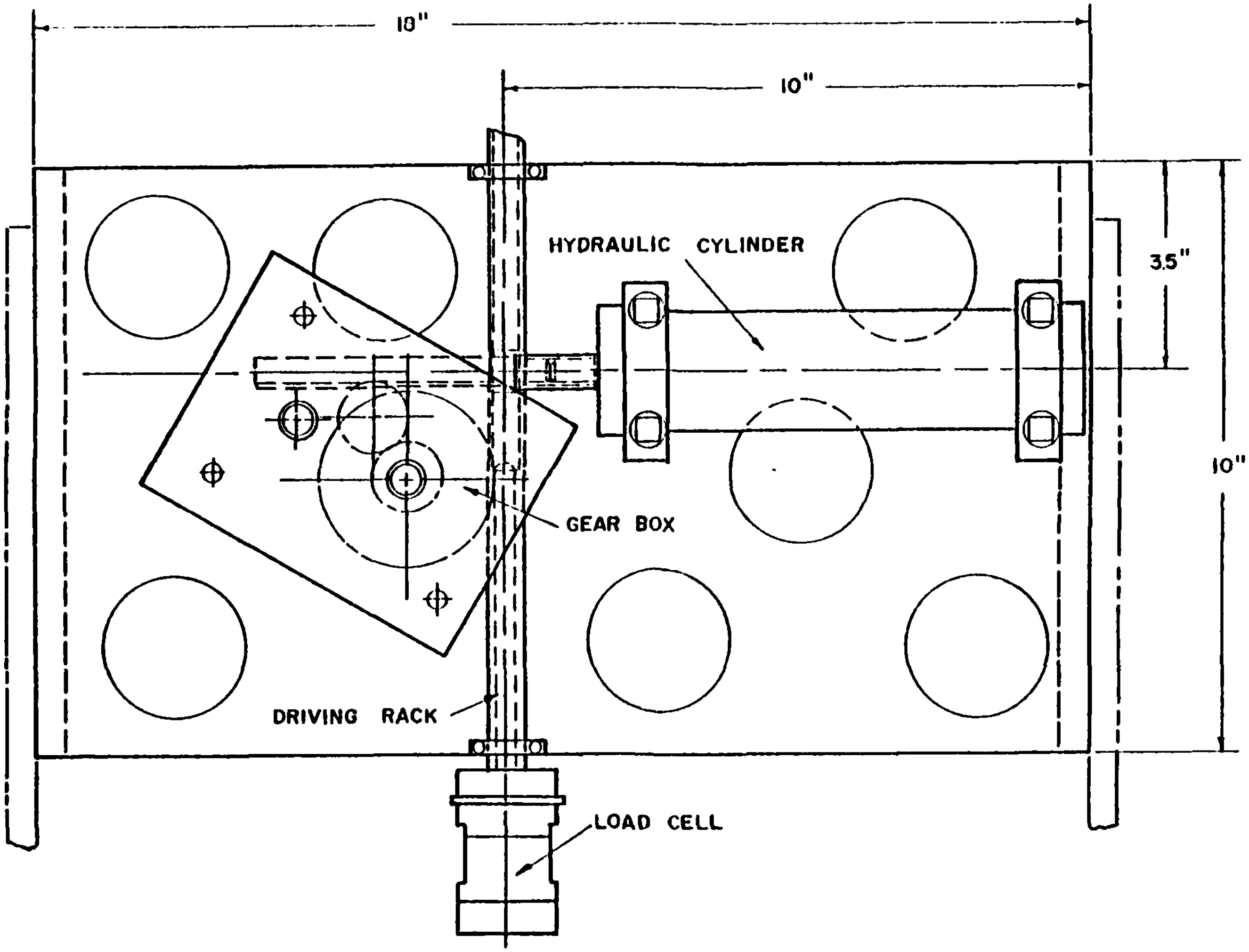


Fig. 8. Driving and loading mechanism

had a stroke of 3-1/8". The vertical stroke of the rack was increased to 6" by the gear system. Two different vertical racks were used. The rack used for driving the piles had a 3/8" spherical radius milled out of its end to allow for interface with the top of the pile. The end of the rack used for the load test was male threaded so that the load cell could be attached.

Hydraulic system

Pressure was supplied to the actuating cylinder by a two-stage hydraulic pump. For the small displacements during the load testing, a pressure generator was used. A three position valve permitted both advancement and retraction of the cylinder.

Load and displacement measurements

Loads were measured with a Schaevitz 1000-pound capacity tension-compression load cell. An aluminum cap was made to attach the load cell to the rack.

Displacements were measured by a Schaevitz 500 MHR Linear Variable Differential Transformer (LVDT). The LVDT was fixed to the side of the driving head and the core was attached to the load cell via an extension rod. This arrangement measured displacement of the load cell rather than of the pile. This was necessary because the pile models were too small for an LVDT core to be attached directly to them without introducing significant lateral forces.

Signals from both the load cell and the LVDT were conditioned by Schaevitz signal conditioners mounted on the centrifuge arms. The power supply for these devices was located outside the centrifuge, with the power and the output signals passed via the slip rings.

The signal was recorded on an X-Y recorder, with the load plotting on the Y-axis and displacement on the X-axis. The load cell was calibrated to the recorder by using dead weights in compression. The LVDT was calibrated by a dial gage. Both calibrations were conducted in 1-g environment.

Video equipment

The tests were observed at the control box over a closed circuit television system. The camera was mounted on the arm very near the central axis. The signal was carried through slip rings to a black and white monitor. A video cassette recorder was also available for use.

Soil

Parent soil and gradation

The soil used for this investigation is one of several that have routinely been used at the University of Colorado. The parent soil is a silica-base sand purchased from a local supplier. The sand is sieved and then blended to the required gradation (Figure 9) to maintain constant engineering properties.

Densities

Three different densities were used in the test program: 108.5 pcf, 109.5 pcf and 110.5 pcf. The soil was placed in the molds in six equal (by weight) lifts, and each lift was rodded to a thickness of 2". Rodding was chosen over tamping because tamping tends to cause surface compaction of each layer and stratification of the specimen. In determining the weight of soil necessary to achieve a required density,

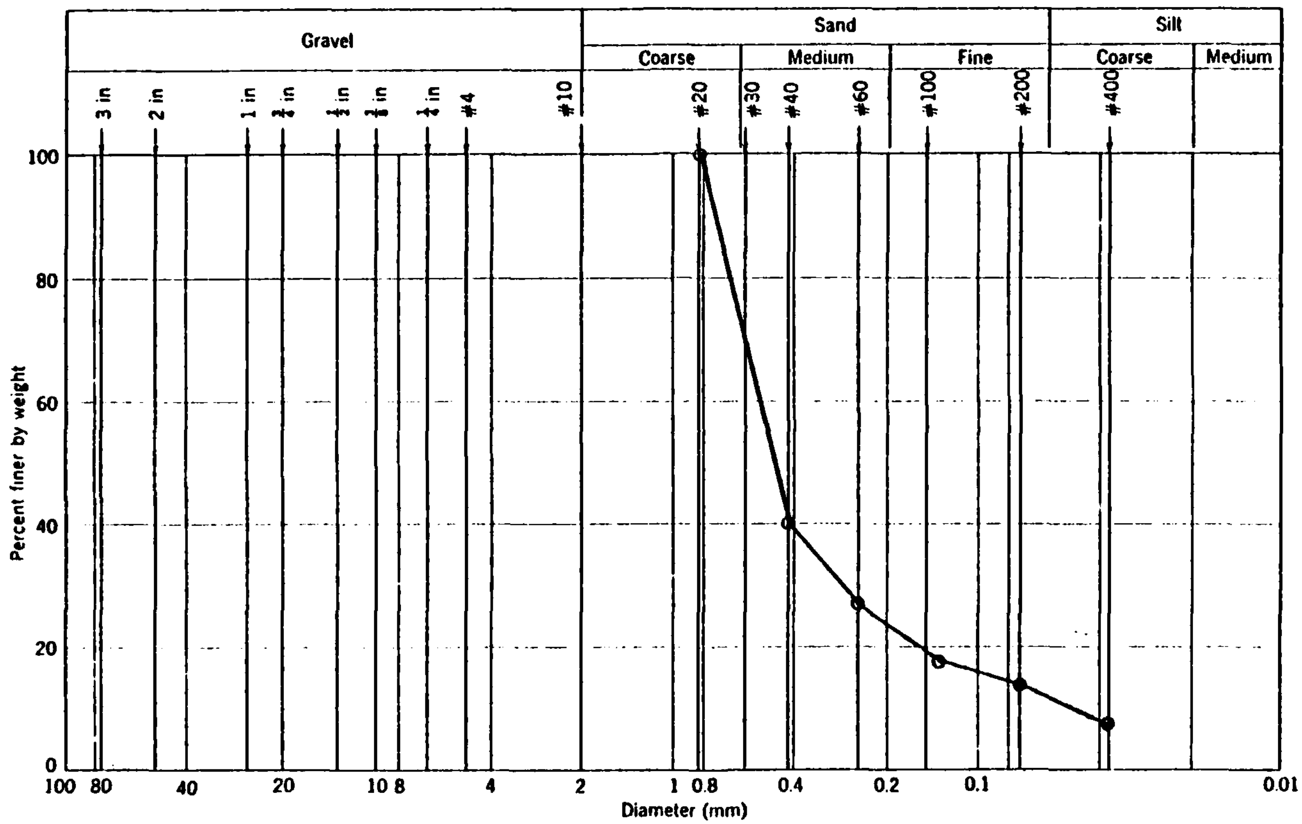


Fig. 9. Gradation of test material

allowance was made for the compaction caused by spinning in the centrifuge. The amount of this compaction varied with the density being sought, but was essentially the same for each of the three centrifuge speeds.

Engineering properties

The strength of this soil has been determined from conventional triaxial shear tests conducted by Kim (1980). By extrapolating from Kim's data, the friction angle, ϕ, was found to be 43.1° at 108.5 pcf, 43.5° at 109.5 pcf, and 44.5° at 110.5 pcf.

CHAPTER V

TEST PROGRAM AND RESULTS

Program

Number of tests

To verify the similitude relationships, the three different sizes of each type of pile were tested at gravity levels inversely proportional to their scales to the prototype, i.e., 50 g, 70 g, and 100 g. Since there is no actual prototype test with which to compare the model results, these three scales give a "model of models" comparison by which to verify the procedure.

As stated earlier, the effect of taper angle was investigated using a prismatic pile and piles with three different tapers. Each pile was tested in three densities of soil. A total of eight tests were repeated to insure the reproduceability of results. The prismatic and .14 in/ft taper piles were retested at 50 g for each of the soil densities. In addition, the 70g tests of the prismatic pile in 109.5 pcf and the .14 in/ft pile in 110.5 pcf were repeated. Because of the poor control of the hydraulic system over the load cell at high speeds, useful results for tests at 100 g were obtained for only seven of the twelve density-taper conditions. The total number of useful tests was 39.

Test procedure

The soil samples were prepared as described in chapter IV and

installed in the centrifuge. The pile model was then attached to the driving device, with the tip resting on the sample and the interface ball at the top in contact with the driving rack. The centrifuge was spun to the appropriate speed and the pile hydraulically pushed into place. The centrifuge was then stopped in order to install the load cell and LVDT, after which the machine was again spun. The load cell was held out of contact with the pile until the desired speed was reached. At this point the X-Y recorder was zeroed to account for the self-weight of the load cell and any electronic variances due to the speed, and the pile was loaded to failure. The load and displacement signals were plotted by the recorder as the loading occurred.

Results

Conversion to prototype

The load tests resulted in 39 charts in three different scales. An example chart is shown as Figure 10. For the data to be fully useful, it was necessary to convert the load penetration curve to some convenient scale, preferably to prototype scale, so that the different tests can be compared. Through use of a Hewlett-Packard 9830 Calculator with plotter, all curves were transformed to prototype scale. The conversion is accomplished by expanding the force scale by n^2 and the displacement scale by n, according to the similitude laws in Table I. Figures 11-22 are the converted curves grouped by pile taper and soil density. The number at the end of each curve indicates the g-level at which the test was run to obtain that curve. The data points for input to the calculator were manually read from the actual test curves. Had

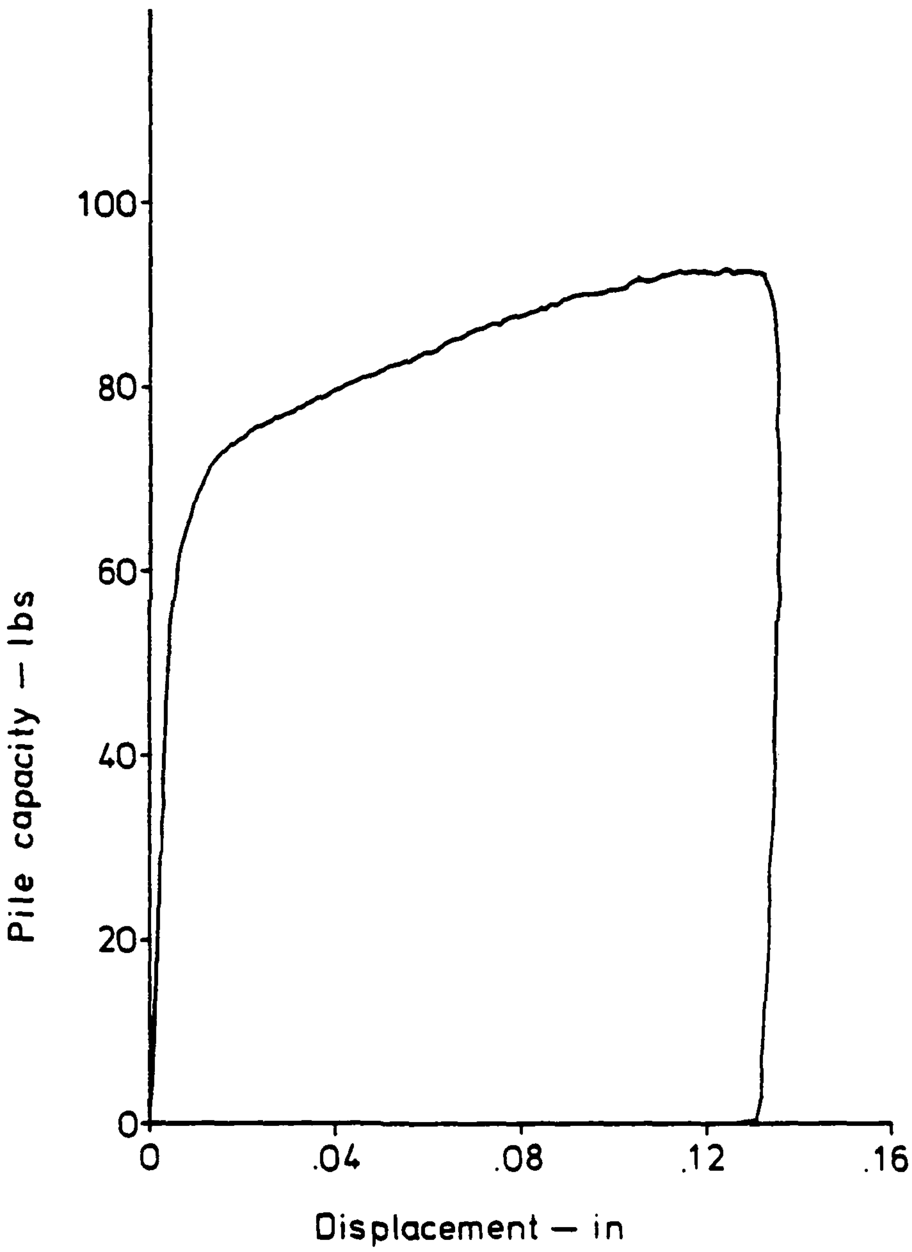


Fig. 10. Example load-displacement curve (results of tests on prismatic pile at 70 g in soil density of 110.5 pcf)

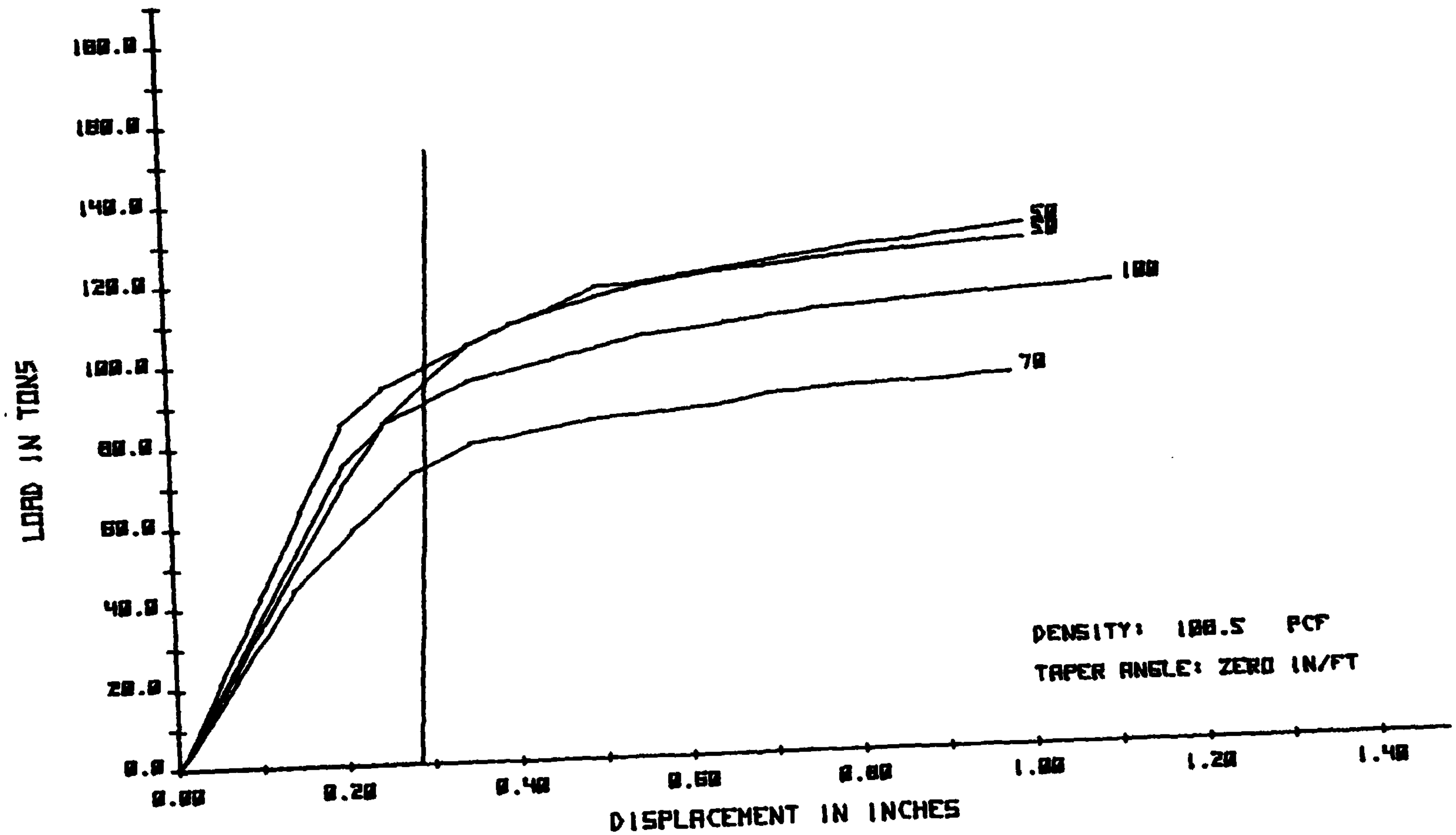


Fig. 11. Prototype-scaled results of tests on prismatic pile, soil density 108.5 pcf.

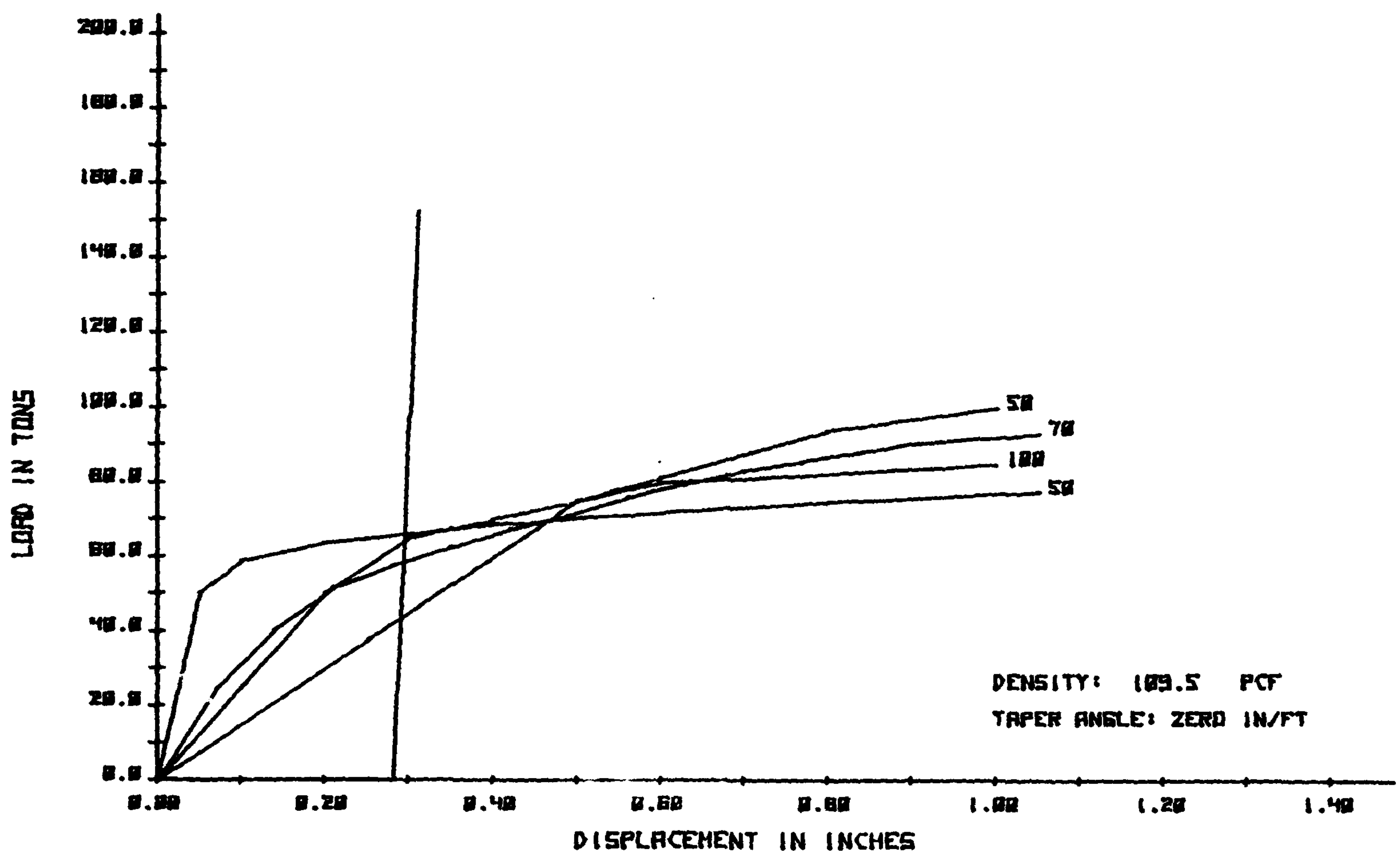


Fig. 12. Prototype-scaled results of tests on prismatic pile, soil density 109.5 pcf.

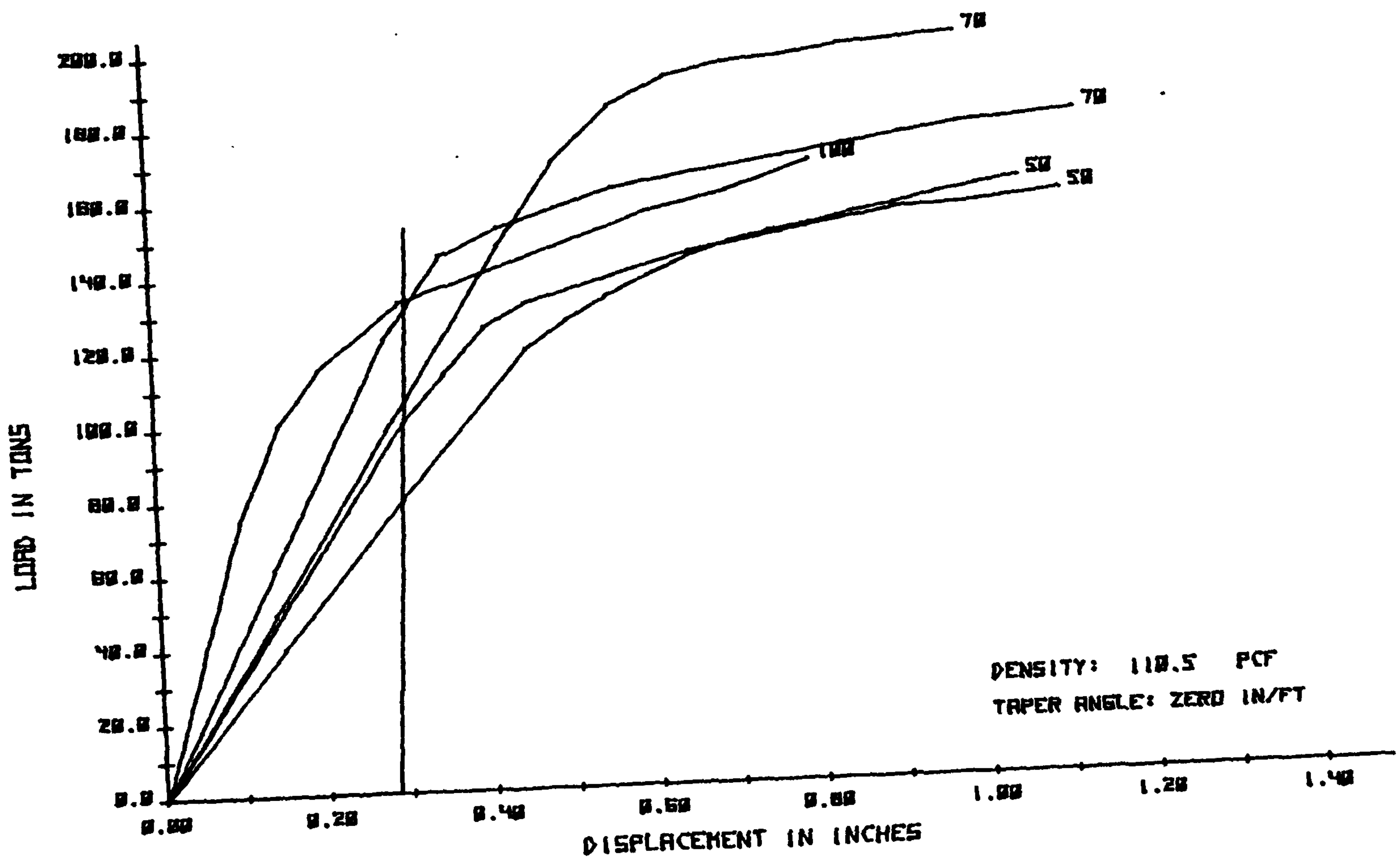


Fig. 13. Prototype-scaled results of tests on prismatic pile, soil density 110.5 pcf.

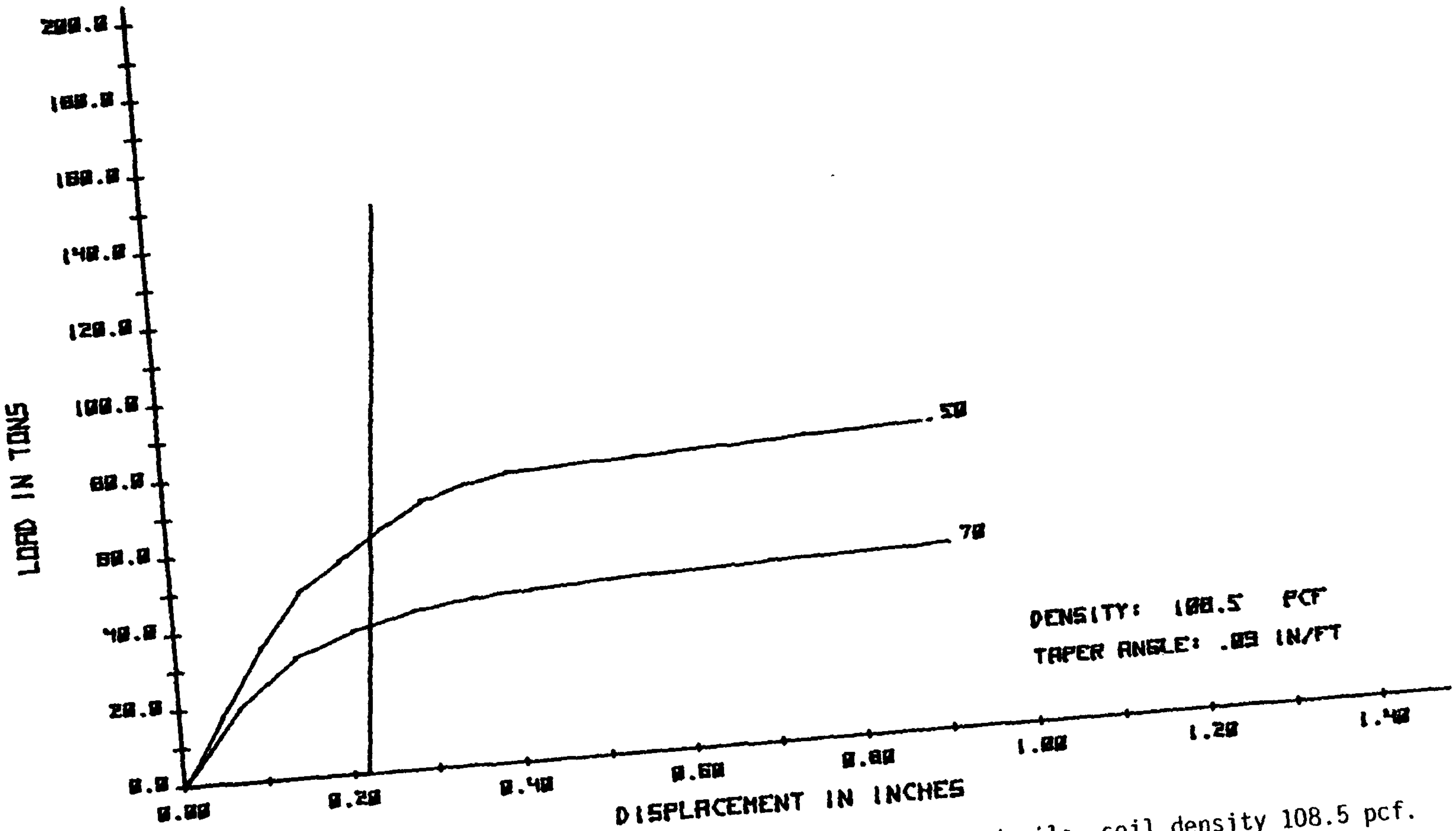


Fig. 14. Prototype-scaled results of tests on .09 in/ft tapered pile, soil density 108.5 pcf.

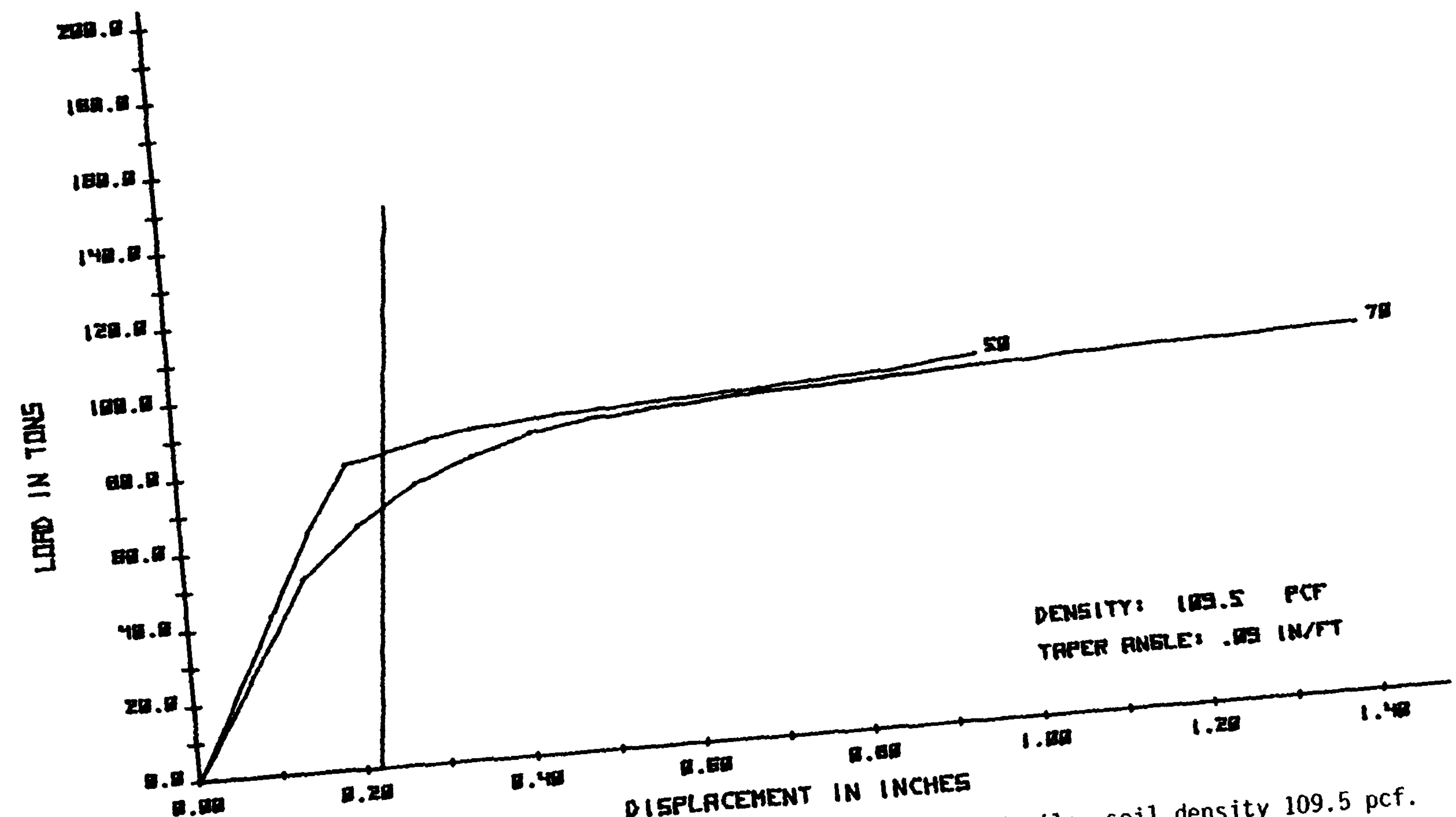


Fig. 15. Prototype-scaled results of tests on .09 in/ft tapered pile, soil density 109.5 pcf.

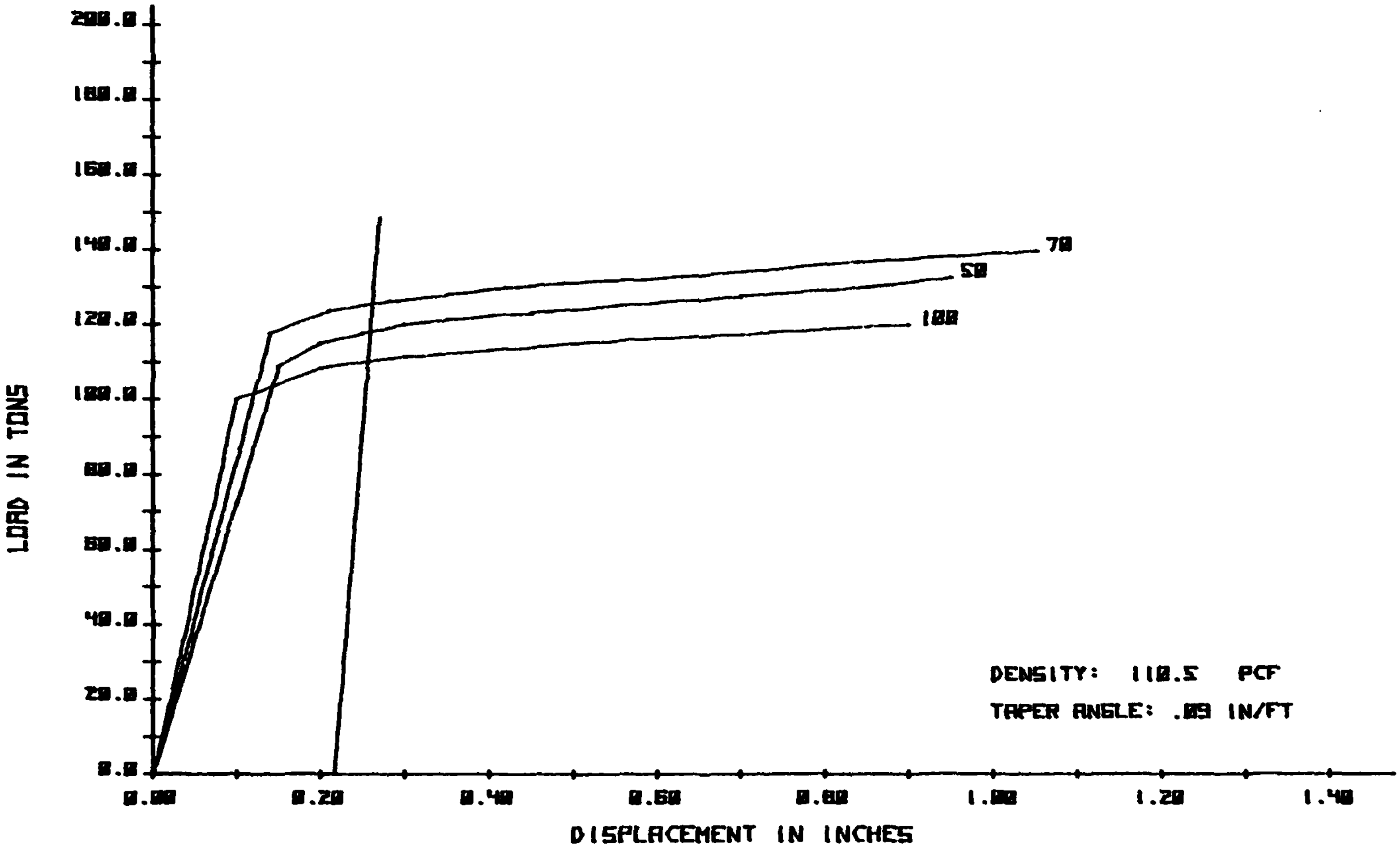


Fig. 16. Prototype-scaled results of tests on .09 in/ft tapered pile, soil density 110.5 pcf.

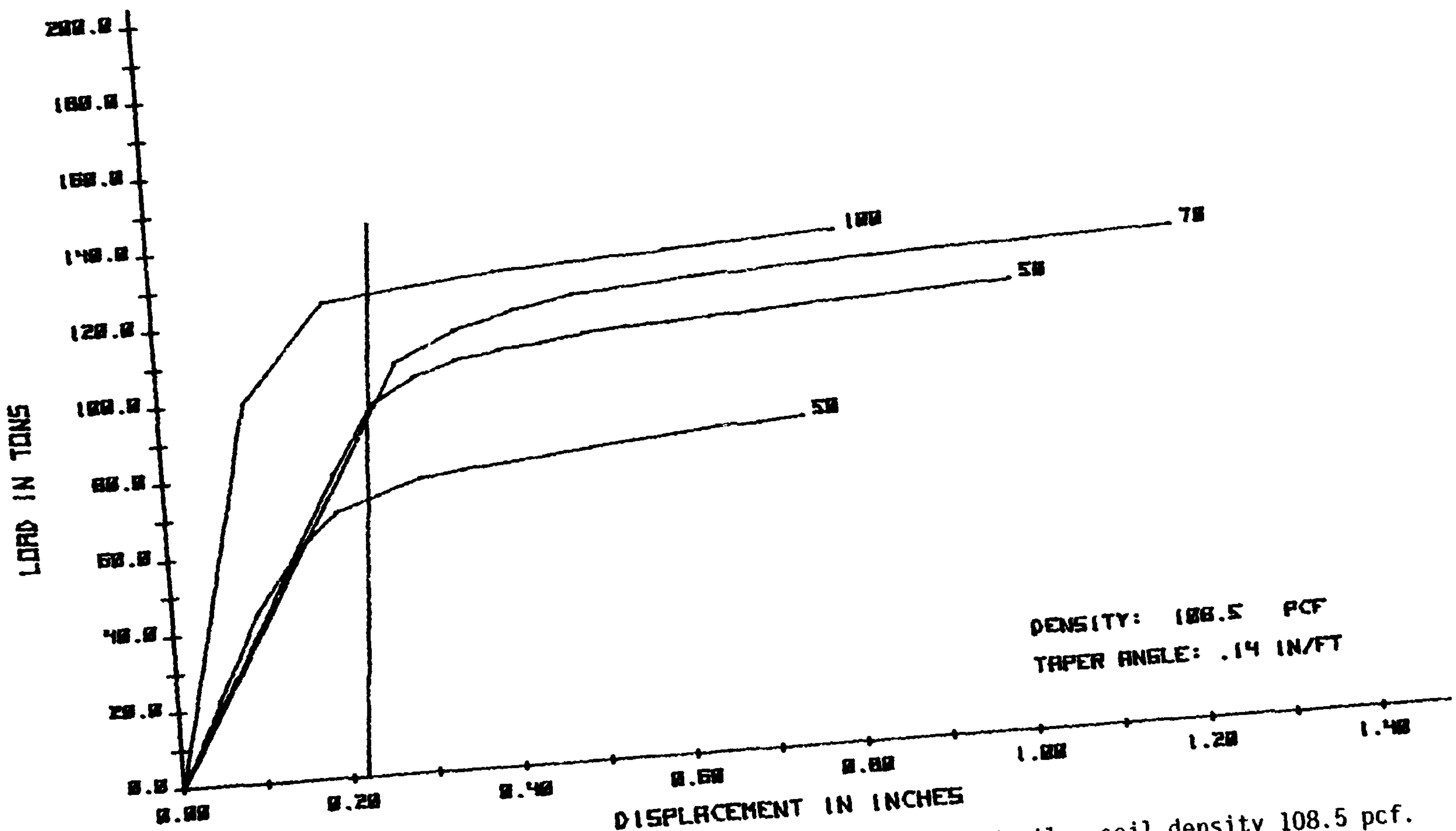


Fig. 17. Prototype-scaled results of tests on .14 in/ft tapered pile, soil density 108.5 pcf.

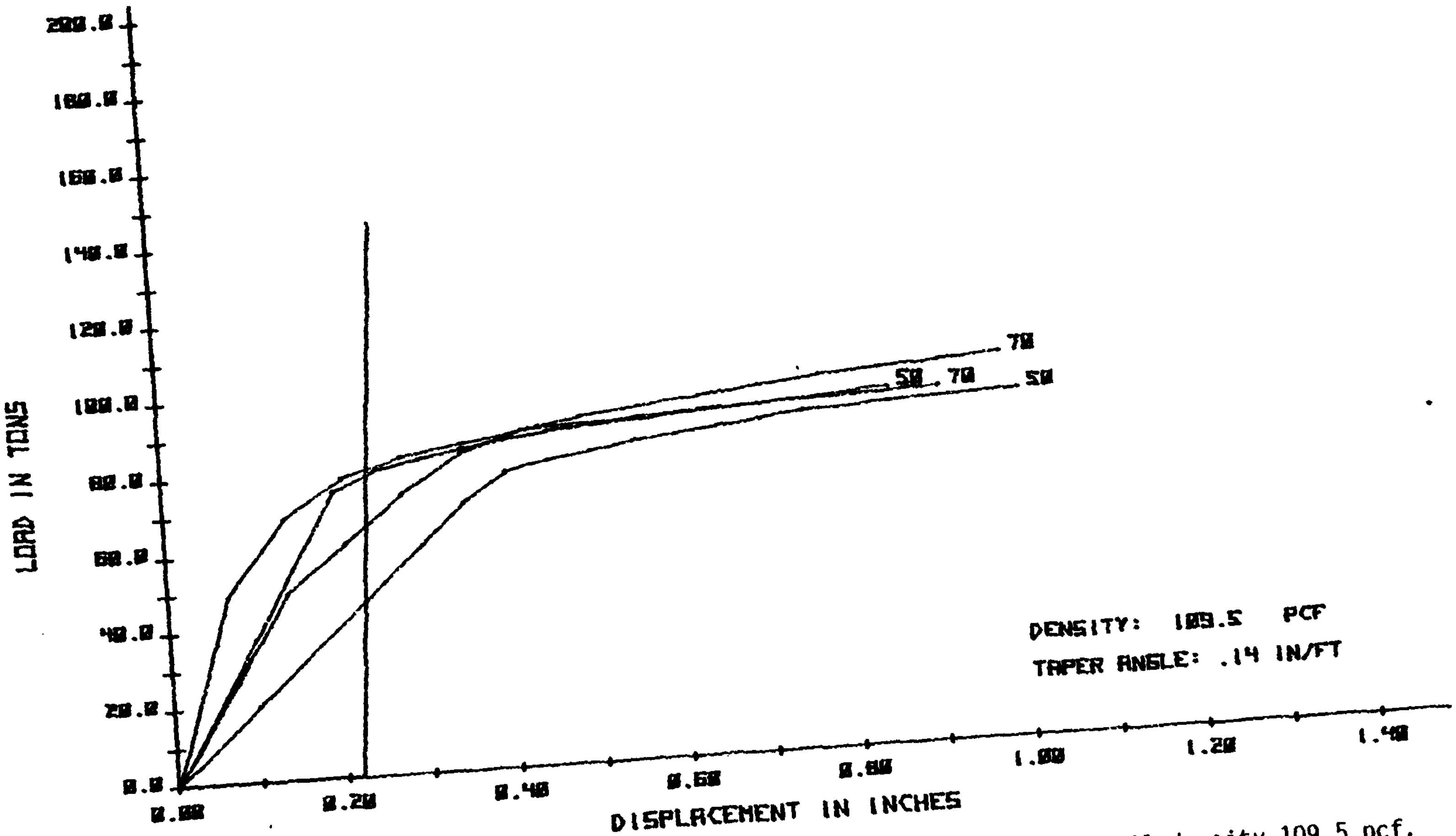


Fig. 18. Prototype-scaled results of tests on .14 in/ft tapered pile, soil density 109.5 pcf.

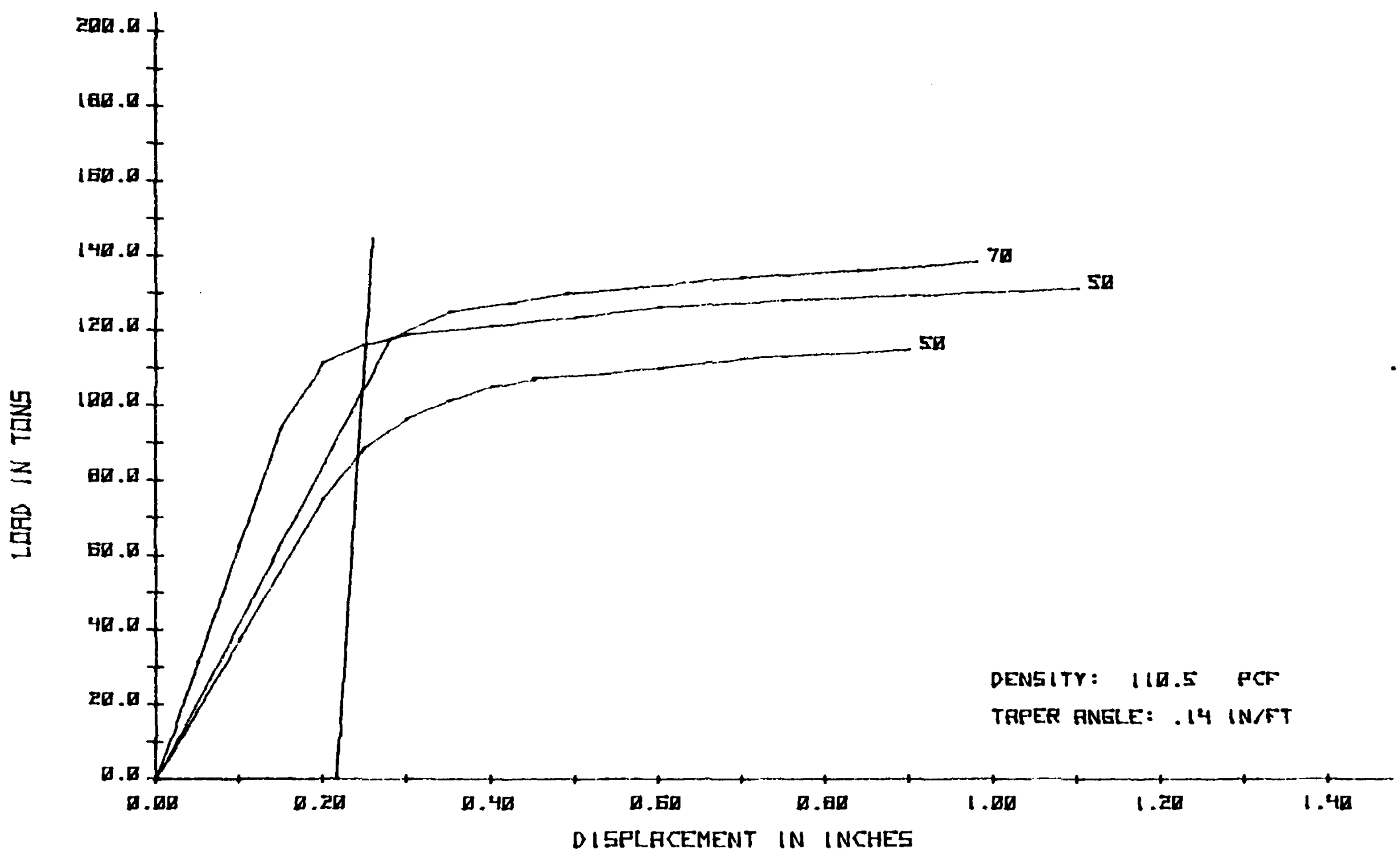


Fig. 19. Prototype-scaled results of tests on .14 in/ft tapered pile, soil density 110.5 pcf.

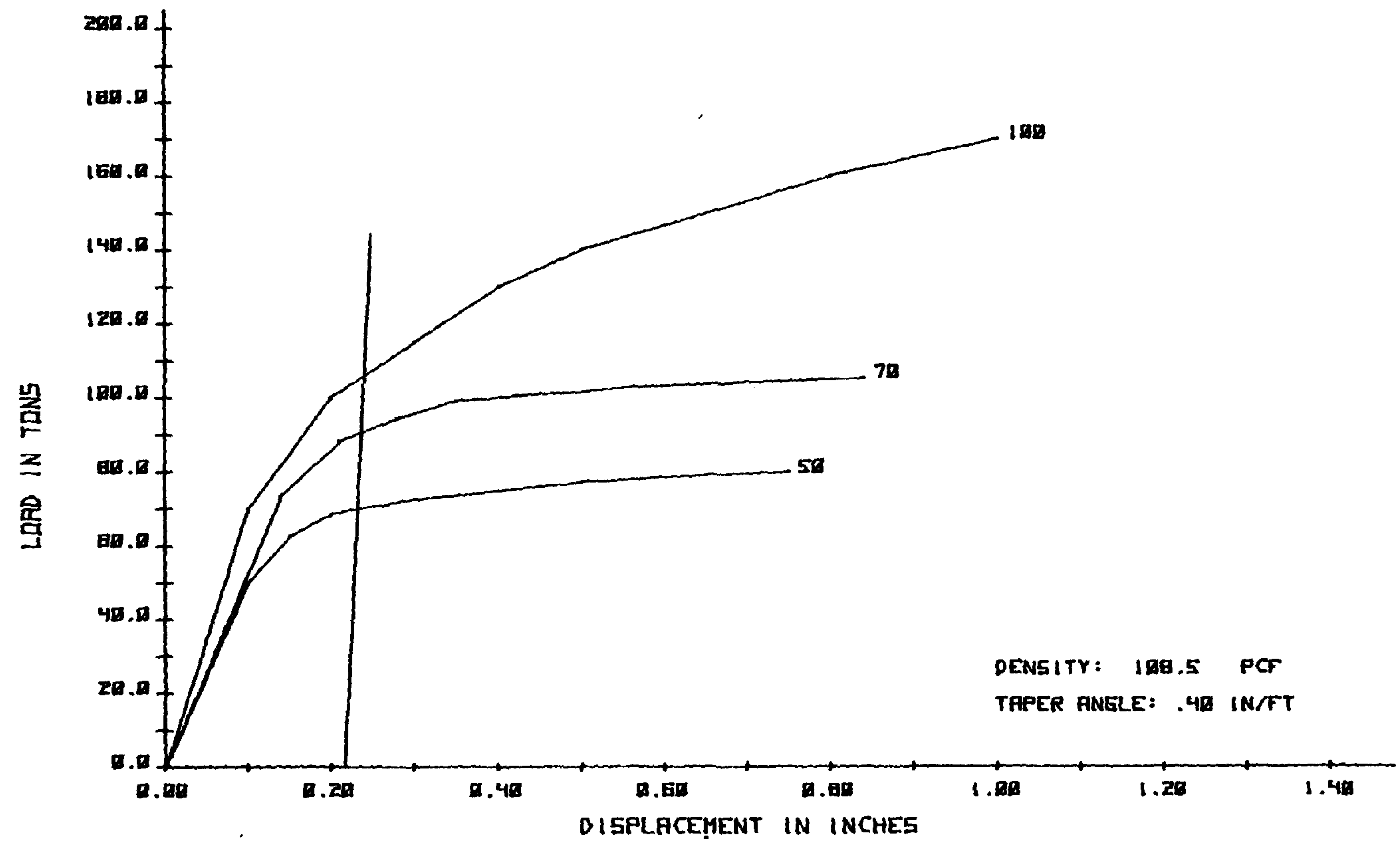


Fig. 20. Prototype-scaled results of tests on .40 in/ft tapered pile, soil density 108.5 pcf.

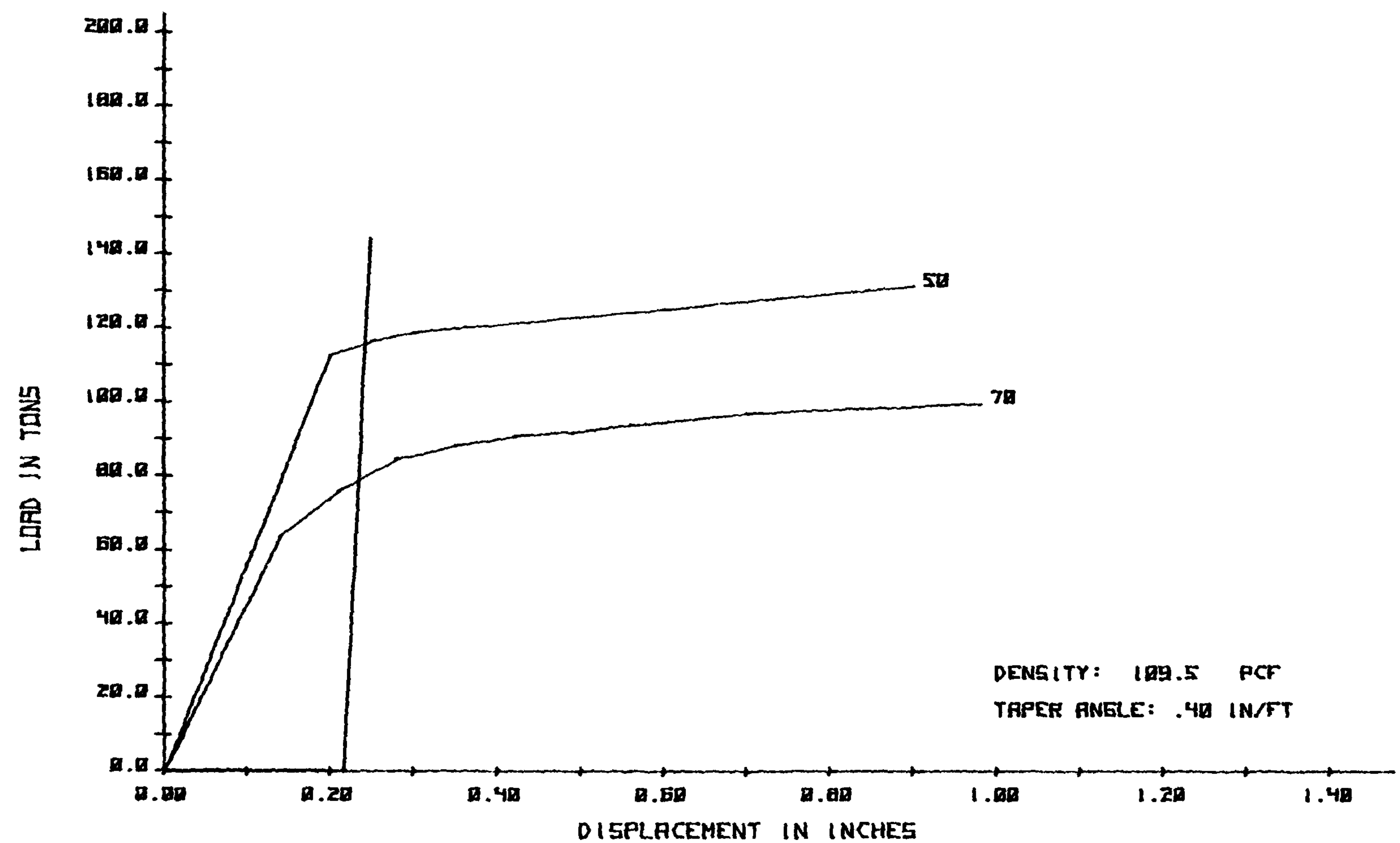


Fig. 21. Prototype-scaled results of tests on .40 in/ft tapered pile, soil density 109.5 pcf.

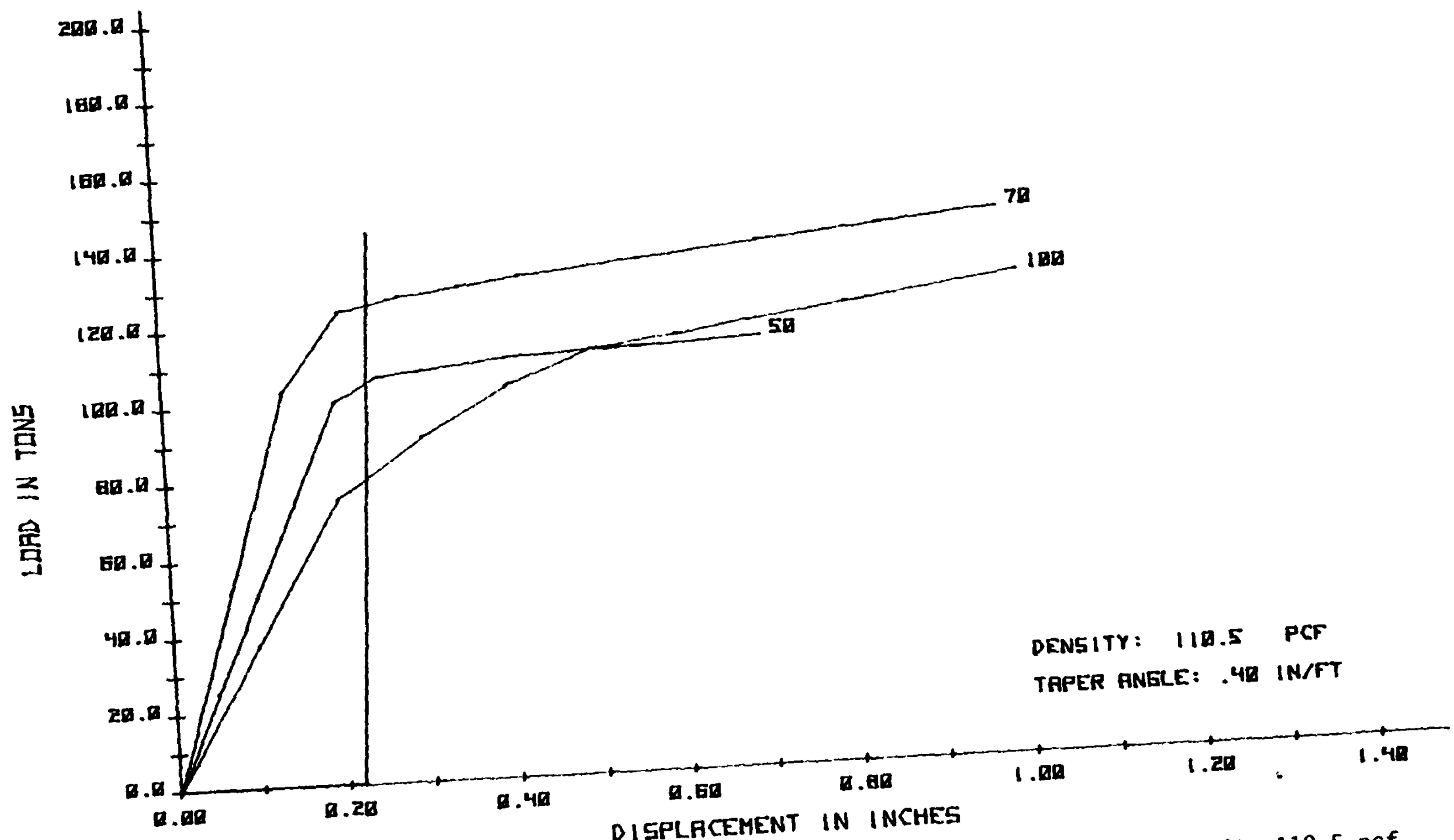


Fig. 22. Prototype-scaled results of tests on .40 in/ft tapered pile, soil density 110.5 pcf.

this method of interpretation been anticipated, the signals from the load cell and LVDT could have been fed to an analog-to-digital converter and recorded on magnetic tape. The tapes could then be fed directly into a conversion program. The HP 9830 has the integral capacity to read digital cassette tapes.

Determination of pile capacity

Davisson's criterion, as discussed in Chapter III, was used to determine the ultimate load. The expression for pile stiffness, K, however, is valid only for the prismatic pile. The stiffness of a tapered pile can be found from statics by integrating strain over the pile length. By this method, K for a pile with a section of constant taper is found to be:

$$K = \frac{E}{\left(\frac{\ell_P}{A_0} + \frac{4\ell_T}{d_0(d_0-d_1)\pi}\right)}$$

where E = modulus of elasticity of the pile

ℓ_P = length of pile without taper

ℓ_T = length of tapered section of pile

A_0 = area of untapered section of pile

d_0 = maximum pile diameter

d_1 = minimum pile diameter

Table IV shows the values thus obtained for the various tapers in prototype scale. In Davisson's criteria the overall pile length is used. For this investigation prototype penetration is 45 ft, with an average overall length of 46.65 ft assumed.

TABLE IV

Pile stiffnesses

TAPER ANGLE (in/ft)	STIFFNESS, K, (kips/in)
0	10775
.09	5485
.14	6682
.40	8873

The offset, δ_A for the prismatic pile was calcualted as 0.283 inches. The tapered piles all have the same prototype diameter, one-half the top diameter. Thus δ_A for the tapered piles is 0.217 inches. Since all load-displacement curves were converted to prototype, the Davisson lines were plotted on Figures 11-22.

CHAPTER VI

ANALYSIS

Verification of Similitude

In a test involving centrifugal models, it is necessary to insure that the laws of similitude have been satisfied in constructing the models and conducting the tests. In this investigation, similitude can be verified by comparison of the tests on different scales sizes using the same density and taper angle. In Figures 11-22, if all curves for a given density and taper were coincident when projected to prototype values, the laws of similitude would be shown to be exactly satisfied.

As can be seen, there is considerable scatter in the prototype-scaled load-displacement curves; nevertheless, the similarity and closeness of the curves lead to the conclusion that the test conditions did satisfy similitude. It will be necessary to investigate further to determine the source of the scatter and its effect on the usefulness of the data. For this latter purpose, the ultimate loads will be compared with respect to taper angle and to density.

The value of ultimate load for each test was taken as the intersection of the load-displacement curve with the line based on Davisson's criteria. These values were averaged for each density-taper condition, with the average ultimate load shown in Table V.

TABLE V

Ultimate loads of test piles

TAPER ANGLE (in/ft)	DENSITY (pcf)	108.5	109.5	110.5
Prismatic		90	58	110
.09		51	77	118
.14		98	68	102
.40		88	98	103

Comparison of Behavior by Soil Density

One measure of the usefulness of the data is a comparison of average failure loads versus soil density. It would be expected that for each taper angle, the ultimate load would increase as a function of increasing soil density. The ultimate load for each density has been plotted as Figure 23, with the points corresponding to each taper angle connected by a straight line. As can be seen, no consistent pattern of load change due to density can be seen. The straight lines connecting the points serve only to identify the different tapers, and are not intended to indicate any sort of correlation.

With the exception of two data points, the results for each density are closely grouped, lending some credence to the accuracy of the data. Such credence, however, is quickly refuted by the sharp drop in values of ultimate load for the medium density.

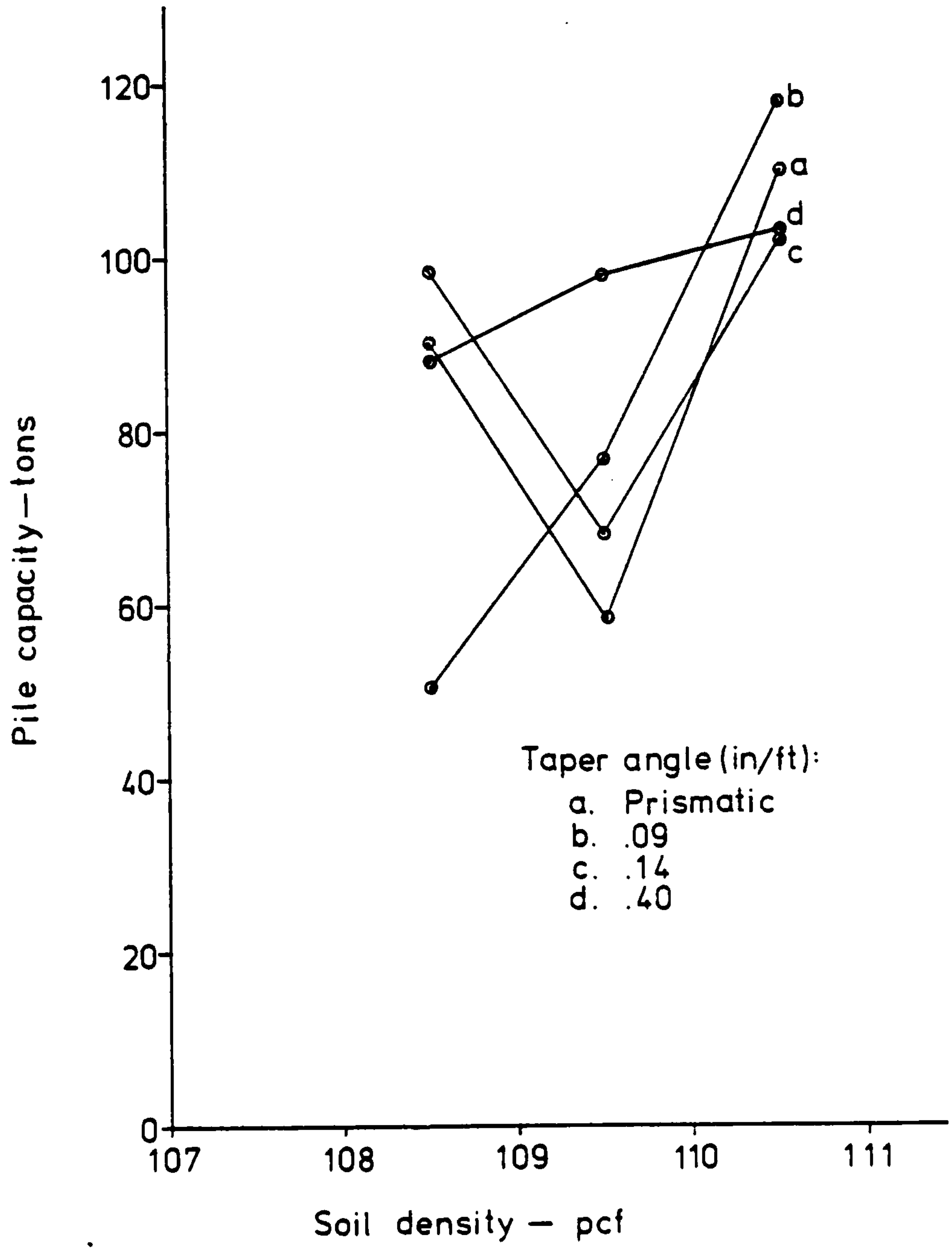


Fig. 23. Pile capacity versus soil density

Comparison of Behavior by Taper Angle

A second measure of usefulness is the comparison of pile capacity with taper angle. If the postulate generally accepted by the piling industry is correct, all other conditions being equal, the capacity will increase with taper angle, or at least peak at some optimum value of taper. The test results have been shown in Figure 24 as a plot of load versus taper angle. This time the values for each density have been connected. Again there is no recognizable pattern suggesting a definitive behavior.

Test Procedure Improvements

The above analysis of the results suggests that the procedure and theory are essentially sound, but that the procedure needs to be further refined to provide more useful results. Several potential sources of inconsistency have been identified, and each will be dealt with individually. These areas are:

1. Uniformity of soil density
2. Grain size effects
3. Container boundary effects
4. Depth of penetration
5. Time between driving and loading
6. Rate of penetration

Uniformity of soil density

Soil density of the specimen is, of course, one of the key variables which must be controlled throughout the test. Obtaining samples

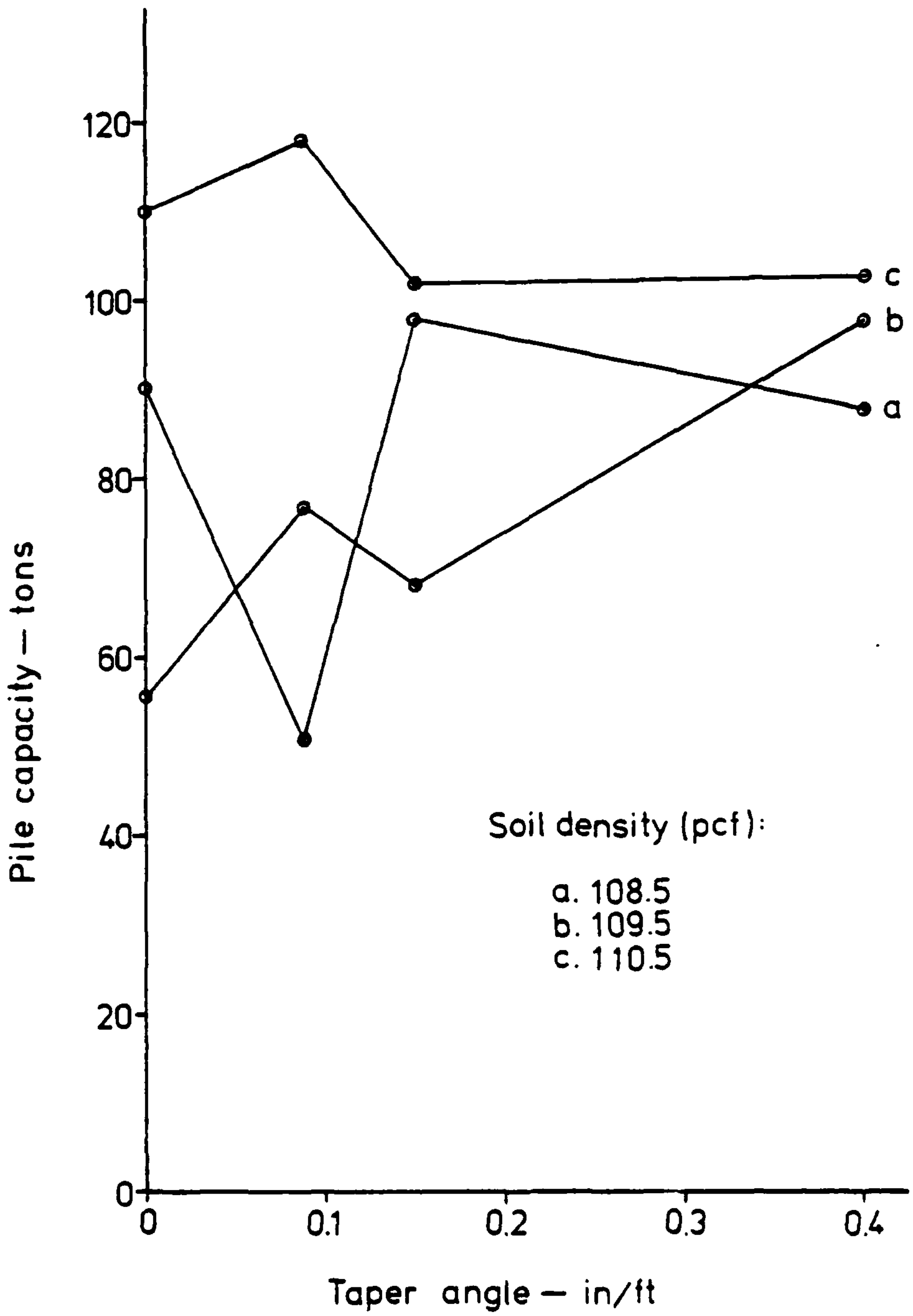


Fig. 24. Pile capacity versus taper angle

which are uniform in density throughout the model and are identical to each other is a difficult task. The method of preparation used in this investigation by rodding of equal lifts to a specified volume is modified from the tamping methods used previously at the University of Colorado. While there is the possibility of variation of density throughout the specimens, the further compaction resulting from handling and centrifugal rotation would tend to reduce the variations. The fact that the final density of the soil after testing was very uniform among models is an indication of the uniformity of the density within each specimen.

Grain size effects

The largest grains in the test material passed a U.S. Standard #20 sieve, which has an opening of .0331 inches, which has to be compared to the smallest pile diameter tested (0.16 in.). For the 100 g tests, the largest particles would appear to the prototype (16") pile as 3.31 inch rocks. This effect may be significant in the results, but would most likely show up as a consistent variation among the three scales. Grain size effect would not, therefore, be a prime contributor to the scatter of the data.

Container boundary effects

The 50 g model provided the most opportunity for boundary effects. The distance between the pile and the side of the container was more than ten pile diameters, which should not have been significant in the results. The 50 g pile came within four pile diameters of the floor of the container, which could have increased load capacity somewhat. The

boundary effect would manifest itself as a consistently higher capacity for the 50 g tests as compared to the 70 g and 100 g tests. Again, since such consistency is not apparent, container effects can neither be definitely excluded nor included. Since the container depth is the maximum practical on this particular centrifuge, it would be necessary to use either models smaller than 50 g or prototypes shorter than 45 feet to avoid such effects.

Depth of penetration

The depth of driving was controlled by a device which stopped the movement of the horizontal rack. While this method appeared to be reliable, a telltale affixed to the driving apparatus which would indicate the depth of penetration and could be viewed via the closed circuit television may provide more accuracy. Similarly, an LVDT might be used for this purpose.

Time between driving and loading

In static load tests of service piles, it has been noted that load capacity is affected by time. This phenomena is generally attributed to consolidation time and dissipation of excess pore pressures. In this investigation there was not only an uncontrolled time lapse between driving and loading, there was also a change in gravity conditions while the load cell was installed. The time between driving and loading can be more closely controlled, but the present test setup dictates that the machine be stopped to insert the load cell. It will be necessary to alleviate the latter problem before the former can be investigated more thoroughly.

The ideal driving system would be one in which the load cell were in place during driving, or in which the load cell could be put in place in flight. If it were in place during driving, then the driving forces could also be analyzed. The load cell used was too big to fit high enough into the driving mechanism to properly drive the pile. A commercially available load cell which would meet the size requirements was not found, and attempts to fabricate one were not successful. Should such a system be developed, the effect of time between driving and loading could easily be investigated.

Rate of penetration

In these tests, the rate of loading was neither controlled nor recorded. As can be seen from Figure 4, the rate of penetration can have a significant effect on the ultimate load, particularly when predicted using Davisson's criteria. These load tests were neither maintained load nor constant rate of penetration tests, though they more closely resembled the latter. The author feels that failure to control rate of penetration during the load tests is probably the most significant reason for the scatter.

The system used for the load tests in this investigation did not allow for good control of the penetration rate. Indeed, for the 100 g test it was extremely difficult to load the pile slowly enough for the recording system to respond. This was the reason so many of the 100 g tests were not usable.

To correct this deficiency, at least two things will be necessary. First, a system must be devised in which the minute control of the loading device is achieved. A load cell which weighs on the order of

1/10 pound would be a step in this direction. The hydraulic system has some inherent flaws which make exact control unlikely. Perhaps an electrically driven system would be more useful. Second, a means of measuring penetration rate need be devised. This can be accomplished by using a chart recorder having time as one of its axes. For this to be successful, time must be properly modeled according to the laws of similitude.

Comparison of Results with Predicted Prototype Behavior

Since no prototype of this particular pile and soil condition is available, the only prototype value with which to compare the model results is a capacity calculated using soil properties. The load capacity calculated in Chapter III for the medium density ranged from 168 tons to 2075 tons. The tests results are somewhat smaller than the lower of the calculated values. Future tests with instrumentation to determine the relative contributions of skin friction and end bearing could be useful in examining such factors as friction angle, bearing capacity factor, and the coefficient between vertical and lateral earth pressure.

CHAPTER VII

CONCLUSIONS

The results of these tests show that centrifugal modeling is definitely a viable tool in investigating axial pile behavior, and they raise some interesting questions for research.

The following recommendations are made for further studies using this technique:

1. The improvements outlined in Chapter VI should be applied to the system. If the equipment can be modified to eliminate the problems arising from the time lapse between driving and loading and from the uncontrolled rate of penetration, it is felt that extremely useful results will be obtained.
2. A wider range of soils and densities should be used. It is difficult to isolate bad data points from trends when only three different densities are used. A variety of soils will provide valuable information on the effect of friction angle on the pile capacity.
3. A better model of the pile should be made. In this study, the pile was solid steel, which is very stiff compared to piles normally used. Models constructed of tubing or less stiff material might be more satisfactory.

4. The analog data should be recorded for conversion to digital tape. This will allow a much more accurate transformation to prototype scale.

In addition to the problems discussed previously, one must also consider the possibility that the great variability of the results obtained was due simply to that type of unpredictable behavior that is seen in actual piling operations.

Some suggested topics for further research are:

1. The effect of time lapse between driving and loading.
2. The effect of rate of penetration during load testing.
3. The relationship between friction angle and load capacity.
4. Load capacity in saturated soils.
5. Modeling of grain sizes.
6. Group pile behavior.

BIBLIOGRAPHY

American Society for Testing of Materials, "Performance of Deep Foundations", ASTM STP 444, 1968.

Bucky, P.B., "The Use of Models for the Study of Mining Problems", Transactions of the American Institute of Mining Engineers, n. 96, 1931.

Bucky, P.B., A.G. Solakian, and L.S. Baldwin, "Centrifugal Method of Testing Models", Civil Engineering, American Society of Civil Engineers, v. 5, n. 5, 1935.

Cargill, K.W., "Centrifugal Modelling of Transient Water Flow in Earth Embankments", M.S. Thesis, University of Colorado, 1980.

Chellis, R.D., Pile Driving Handbook, Pittman Publishing Co., New York, 1944.

Chellis, R.D., Pile Foundations, McGraw-Hill Book Co., New York, 1951.

Davisson, M.T., "High Capacity Piles", Proceedings Lecture Series, Innovations in Foundation Construction, ASCE, Illinois Section, 1972.

Fedorov, I.S., "The Investigation of Deformations in Foundations by Means of Models", Technical Physics of the USSR, v. 3, n. 5, 1936.

Fellenius, B.H., "Pile Foundations - Analytical", Analysis and Design of Building Foundations, H.Y. Fang, ed., Envo Publishing Co., Lehigh Valley, Pennsylvania, 1976.

Ferguson, K.A., "Centrifugal Modeling of the Quasi-Static Cone Penetrometer", M.S. Thesis, University of Colorado, 1980.

Goble, G.G., and G.E. Likins, Jr., "A Report on a Static and Dynamic Pile Test in West Palm Beach, Florida", Department of Solid Mechanics, Structures, and Mechanical Design, Case Institute of Technology, Cleveland, Ohio, 1973.

Goble, G.G. and F. Rausche, "Wave Equation of Pile Analysis", Federal Highway Administration Report 76-14, 1976.

Kim, M.M., "Centrifuge Model Testing of Soil Slopes", Ph.D. Thesis, University of Colorado, 1980.

Lambe, T.W., and R.V. Whitman, Soil Mechanics, John Wiley and Sons, New York, 1969.

Meyerhof, G.G., "The Ultimate Bearing Capacity of Foundations", Geotechnique, v. 2, 1951.

Meyerhof, G.G., "Penetration Tests and Bearing Capacities of Cohesionless Soils", Journal of the Soil Mechanics and Foundation Division, ASCE, v. 82, n. SM1, 1956.

Mikasa, M., and N. Takada, "Significance of Centrifugal Model Test in Soil Mechanics", Proc. Eighth International Conference on Soil Mechanics and Foundation Engineering, v. 1.2, 1973.

Mikasa, M., N. Takada, and K. Yamada, "Centrifugal Model Tests of a Rockfill Dam", Proc. Seventh Int'l. Conf. on Soil Mech. and Fndn. Engr., v. 2, 1969.

New York State Department of Transportation Soil Mechanics Bureau, Static Pile Load Test Manual, SCP 4-74, 1974.

Oveson, N.K., "Centrifugal Testing Applied to Bearing Capacity Problems of Footings on Sand", Geotechnique, v. 25, 1975.

Panek, L.A., "Theory of Model Testing as Applied to Roof Bolting", Bureau of Mines Report of Investigations 5154, 1956.

Peck, R.B., "A Study of the Comparative Behavior of Friction Piles", Highway Research Board Report No. 36. 1958.

Pokrovsky, G.I., and I.S. Fedorov, "Studies of Soil Pressures and Soil Deformations by Means of a Centrifuge", Proc. First Int'l. Conf. on Soil Mech. and Fndn. Engr., 1936.

Roscoe, K.H., "Soil and Model Tests", Journal of Strain Analysis, v. 3, n. 1, 1968.

Roscoe, K.H., "Influence of Strains in Soil Mechanics", Geotechnique, v. 20, n. 2, 1970.

Roscoe, K.H., A.N. Schofield, and C.P. Wroth, "On the Yielding of Soils", Geotechnique, v. 8, n. 9, 1958.

Roscoe, K.H., and H.B. Poorooshasb, "A Fundamental Principle of Similarity in Model Tests for Earth Pressure Problems", Proc. Second Asian Conf. on Soil Mech. and Fndn. Engr., Tokyo, v. 1, 1963.

Samson, C.H., T.J. Hirsch, and L.L. Lowery, "Computer Study for Dynamic Behavior of Piling", Journal of the Structural Division, ASCE, v. 89, n. ST4, 1963.

Terzaghi, K., and R.B. Peck, Soil Mechanics in Engineering Practice, 2nd. ed., John Wiley and Sons, New York, 1967.

Vesic, A.S., "Bearing Capacity of Deep Foundations in Sand", Highway Research Board Report No. 39, 1963.

Vesic, A.S., "A Study of Bearing Capacities of Deep Foundations", Georgia Institute of Technology, Atlanta, 1967.

Vesic, A.S., "Ultimate Loads and Settlements of Deep Foundations in Sand", Proc. Symposium on Bearing Capacity and Settlement of Foundations, Duke University, Durham, N.C., 1967.

Vesic, A.S., Principles of Pile Foundation Design, Duke University School of Engineering, 1975.

Whitaker, T., The Design of Piled Foundations, Pergammon Press, 1976.

Winterkorn, H.F., and H.Y. Fang, Foundation Engineering Handbook, Van Norstrand Reinhold, New York, 1975.